MAGNETOHYDRODYNAMICS Vol. 54 (2018), No. 4, pp. 353–360
DOI: 10.22364/mhd.54.4.3

MAGNETOVISCOUS EFFECT IN CASE OF MAGNETIC FLUID OSCILLATIONS IN STRONG MAGNETIC FIELD

V.M. Polunin, P.A. Ryapolov, A.I. Zhakin, A.M. Ivanov, E.V. Shel'deshova

Southwest State University (SWSU), Kursk, Russia
e-Mail: poluninvm1@gmail.com

The paper gives an assessment of the viscosity increment (magnetoviscous effect) in a thin near-wall layer of a column of magnetic fluid which oscillates in a tube when applying a strong transverse magnetic field. The value of the magnetoviscous effect was calculated using a formula derived from two different theoretical approaches and applied to previously published experimental results of complex measurements of the oscillation frequency and saturation magnetization of a magnetic fluid under the assumption that there is no viscosity field dependence.

Introduction. The most well-known applications of nanodispersed magnetic fluids (MFs), which have already become traditional, are magnetic fluid seals, separators of non-magnetic materials, tilt and acceleration sensors, and fillers for loudspeaker magnetic heads [1–5].

In the last decade, researchers' attention is drawn to the application of MFs for solving specific problems of oscillation damping by applying the magnetic field [6–8], of dispensing small portions of gas into reactors as a result of magnetic field modulation by air bubbles oscillating in MF [9–10].

In [11–12], the measurements and theoretical analysis of the oscillations of a magnetic fluid column sustained by magnetic levitation in a tube exposed to a strong magnetic field are presented. Moreover, in [12], the calculations made using a ponderomotive elasticity model with the applied correction for the resistance of a moving viscous fluid are compared with the experimental magnetization curve. However, in the theoretical model, the MF viscosity invariance is assumed to be one of the approximations. Consequently, the problem of magnetoviscous effect, i.e. the viscosity increment in the MF thin near-wall layer under the applied magnetic field, is still unresolved.

In shear flow, a moment of forces acts on a solid particle, which leads to its rotation. The magnetic field aligns the magnetic moment of the particle and, if there is a relationship (interaction) between the moment of the particle and the particle itself, hinders its free rotation. This leads to local gradients of the velocity of the base fluid near the particles and increases the MF effective viscosity [13–17]. Saturation of the so-called rotational viscosity occurs when a strong field rigidly aligns the particles. The actual viscosity increment in strong magnetic fields for a magnetite sample with a volume concentration of $0.19-0.24$ in a Poiseuille flow through a capillary subject to a perpendicular magnetic field does not exceed $5-6\%$ [18]. At present, all known experimental data on complex measurements of the frequency of elastic oscillations of the MF column in a strong magnetic field and saturation magnetization are interpreted basing of an assumption of the absence of such dependence. An exception is an article in the collection of scientific papers [19] which presents preliminary results which need to be additionally verified.

The lack of information on the magnetoviscous effect does not allow obtaining a comprehensive picture of the physical mechanisms of the oscillatory motion of magnetic fluid active elements in various engineering devices and makes difficult the use of the gained methodological experience to expand the means assisting in controlling the wear resistance and consumption in magnetic colloids.

In the present study, in order to expand the physical understanding of the MF oscillatory flow in a magnetic field, we have determined the magnetoviscous effect in a thin near-wall layer under the influence of a strong transverse magnetic field. The value of the magnetoviscous effect was calculated by a formula derived from two different theoretical approaches and applied to the previously published experimental results of the comprehensive measurement of the oscillation frequency and saturation magnetization of magnetic fluid assuming the absence of the field dependence of viscosity [12, 20].

1. Brief review of sample parameters and experiments already published. With reference to the task set, it is necessary to describe briefly the experimental setup and the measurement procedure. The experimental setup designed to measure the MF column oscillation frequency was described in detail in [12] (scheme no 1), and all explanations are given in [20] in the description of the diagram in Fig. 2. In the present work, we consider only the main components of the setup. In the experiment, we used a laboratory electromagnet FL-1; a tube with the internal diameter $d = 12\,\mathrm{mm}$ made of Plexiglas was placed between the pole tips of the electromagnet. The tube axis passes horizontally through the centre of the pole gap and is parallel to the surface of the pole tips. The OZ-axis coincides with the axis of the tube, and its beginning is in the centre of the MF column in the equilibrium state.

A significant detail of the obtained dependences of the transverse value of the magnetic field $H_x(z)$ is the presence of a linear section on the curves at $z = 57.5\,\mathrm{mm}$, which makes it possible to consider the magnetic field intensity gradient in this area $\Delta H_x/\Delta z = \mathrm{const}$. In a strong and non-uniform magnetic field, the MF-column takes a cylinder-like shape. In the conducted experiment, the distance between the bases of the cylinder was $b = 115\,\mathrm{mm}$. In the view of the assumptions, concerning the tube 'thinness', the magnetic field at the points on the MF free surface is tangential to it, i.e. it has only the tangential component H_x (perpendicularly to the surface of the pole tips), and the magnetic field intensity gradient is directed perpendicularly to the surface along the OZ-axis towards its beginning. The maximum magnetic field in the centre between the poles of the electromagnet is $900\,\mathrm{kA/m}$.

In [12], the samples of magnetic fluids MF-1 and MF-2 are described, based on finely dispersed magnetite Fe3O4 which is stabilized by a surfactant (oleic acid $C_8H_{17}CH = CH(CH_2)_7{-}COOH$). In the MF-1 sample, aviation kerosene TS-1 was used as a dispersive medium, i.e. a carrier liquid; in the MF-2 sample, undecane $C_{11}H_{24}$ (alkane class hydrocarbon) was used as a carrier liquid. The objects of the research were synthesized in the Fundamental Scientific Research Laboratory of Applied Ferrohydrodynamics of the Ivanovo State Power Engineering University.

The densities of the MF-1 and MF-2 samples and their saturation magnetization are $\rho = 1245\,\mathrm{kg/m^3}$ and $\rho = 1227\,\mathrm{kg/m^3}$, $M_\mathrm{s} = 39.5\,\mathrm{kA/m}$ and $M_\mathrm{s} = 40.4\,\mathrm{kA/m}$, respectively. The shear viscosity η for MF-1 and MF-2 is $34.8\,\mathrm{mPa{\cdot}s}$ and $30.4\,\mathrm{mPa{\cdot}s}$, respectively, at a shear rate of $79.200\,\mathrm{1/s}$.

2. Theoretical validation. Under the effect of differential pressure on the bases of the MF column with the cross-section S and length b, a small displacement of the centre of mass by δz occurs, which is accompanied by a fluid volume gain at the point with the coordinate $z = b/2$ on $S\delta z$ and by its volume reduction by the same amount at the point $z = b/2$.

Due to the magnetic field symmetry, the expression for the perturbation of the ponderomotive force along the OZ-axis at finite increments Δz and taking into account the fact that $\Delta z \ll b$,

$$\left(\frac{\partial H_x}{\partial z}\right)_{z=-b/2} = -\left(\frac{\partial H_x}{\partial z}\right)_{z=b/2}, \quad (M_x)_{z=b/2} = \frac{(M_x)_{z=-b/2} + (M_x)_{z=b/2}}{2}$$

takes the form

$$\Delta f_z = -2\mu_0 S \left(M_x \frac{\partial H_x}{\partial z}\right)_{z=b/2} \Delta z, \tag{1}$$

where M_x is the magnetization component, normal to the surface of the pole tips.

From Eq. (1) immediately follows the expression for the ponderomotive elasticity coefficient k_p:

$$k_\mathrm{p} = \mu_0 \frac{\pi d^2}{2} \left(M_x \frac{\partial H_x}{\partial z}\right)_{z=b/2}. \tag{2}$$

In the framework of continuum mechanics, i.e. neglecting the dispersion of the system and the interaction of the dispersed particles with each other, the equation of harmonic oscillations reads as

$$M_\omega \frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} + r'' \frac{\mathrm{d}\xi}{\mathrm{d}t} + k_\mathrm{p}\xi = 0, \tag{3}$$

where ξ is the displacement from the equilibrium position of the centre of gravity of the MF column in the tube, r'' is the fluid active resistance on the inner surface of the tube. The expression for r'' is given in [21]:

$$r'' = \pi d b \sqrt{\rho\eta\omega/2}. \tag{4}$$

Eq. (4) for the resistance coefficient was first derived by Helmholtz. The limitation is the ratio of the tube circumference πd to the length of the viscous wave

$$\lambda' = 2\pi \sqrt{\frac{2\eta}{\rho\omega}}$$

at which it exceeds 10. In the case under consideration, at the maximum frequency 15 Hz used in the experiment, this ratio is 7.5.

In Eq. (3), M_ω can be represented as follows:

$$M_\omega = m\left(1 + \frac{2}{d}\sqrt{\frac{2\eta}{\rho\omega}}\right), \tag{5}$$

where m is the mass of the fluid in the tube.

The second term in brackets is relatively small (≈0.1); being multiplied by the mass of the MF, it represents the so-called added mass determined by the fluid viscosity. The added mass is usually neglected [21]. However, in this case, this additive can influence the numerical value of the oscillation frequency with a measured accuracy.

Let us rewrite the equation of damped oscillations (3) in the standard form:

$$\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} + 2\beta \frac{\mathrm{d}\xi}{\mathrm{d}t} + \omega_0^2 \xi = 0, \tag{6}$$

where

$$2\beta = \frac{r'}{M_\omega} = \frac{r'}{m\left(1 + (2/d)\sqrt{2\eta/(\rho\omega)}\right)} = \frac{2\beta_H}{1 + (2/d)\sqrt{2\eta/(\rho\omega)}},$$

$$\beta_H = \frac{1}{d}\sqrt{\frac{2\eta\omega}{\rho}}. \tag{7}$$

The last term in Eq. (7) represents the oscillations damping coefficient obtained from the Helmholtz formula (4). In Eq. (5),

$$\omega_0^2 \equiv k_\mathrm{p}/M_\omega. \tag{8}$$

The general solution of Eq. (5) has the known form $\xi = C\mathrm{e}^{-\beta t}\cos(\omega t + \psi)$, where C, ψ are arbitrary constants, and the oscillation frequency ω is expressed as

$$\omega = \sqrt{\omega_0^2 - \beta^2} \tag{9}$$

After algebraic transformations, Eq. (9) reads as

$$\omega^2 = \frac{k_\mathrm{p}}{m(1 + (d/2)\sqrt{2\eta/(\rho\omega)}} - \frac{\beta_H^2}{\left(1 + (2/d)\sqrt{2\eta/(\rho\omega)}\right)^2}, \tag{10}$$

or as

$$4\pi^2\nu^2 = \frac{4k_\mathrm{p}}{\pi\rho b d^2(1 + (d/2)\sqrt{\eta/(\pi\nu\rho)}} - \frac{\beta_H^2}{\left(1 + (2/d)\sqrt{\eta/(\pi\nu\rho)}\right)^2} \tag{11}$$

Neglecting the second term in the right-hand side of Eq. (11) and taking into account Eq. (2) yield

$$\pi^3\nu^2 d^2 b\rho + 2db\sqrt{\pi^5\eta\rho\nu^3} = \mu_0\frac{\pi d^2}{2}\left(M_x\frac{\partial H_x}{\partial z}\right)_{z=b/2}. \tag{12}$$

There is another approach to solve this problem; it is based on the application of the law of energy conservation in the oscillatory system [12, 20]. In the presence of the dissipation of the oscillation energy, by assuming the kinetic energy value $E_{\mathrm{k}0}$ as maximum at the initial time, the potential energy of the oscillation motion after a quarter of the period can be represented as

$$\frac{k_\mathrm{p}\Delta z_{01}^2}{2} = \frac{k_\mathrm{p}\Delta z_0^2}{2} - \frac{\pi^2 bd}{2}\sqrt{\frac{\omega^3\eta\rho}{2}}\cdot\frac{\Delta z_0^2}{2}. \tag{13}$$

If we make a transition in the left-hand side of Eq. (13) as $\Delta z_{01} \to \Delta z_0$ and $k_\mathrm{p} \to (k_\mathrm{p} + \delta_\eta)$, and assume that the equality is formally satisfied, we get:

$$\frac{(k_\mathrm{p} + \delta_\eta)\Delta z_0^2}{2} = \frac{k_\mathrm{p}\Delta z_0^2}{2} - \frac{\pi^2 bd}{2}\sqrt{\frac{\omega^3\eta\rho}{2}}\cdot\frac{\Delta z_0^2}{2}. \tag{14}$$

In this case, δ_η is the correction for the ponderomotive elasticity coefficient, determined by the viscous fluid flow,

$$\delta_\eta = -\frac{\pi^2 bd}{2\sqrt{2}}\sqrt{\omega^3\eta\rho}. \tag{15}$$

However, the formula derived for δ_η gives some "peak" estimate (the maximum value after a quarter of the period). The viscous elasticity coefficient $k_\eta = (2/\pi)\delta_\eta$ is the half-period averaged value of the harmonic function of the maximum value, therefore,

$$k_\eta = -\frac{2}{\pi}h_\mathrm{f} d\sqrt{\pi^7\nu^3\eta\rho}. \tag{16}$$

In addition, it was presumed previously that the MF viscosity does not depend on the magnetic field intensity. Assuming such dependence, replacing η by η_H yields

$$k_\eta = -2h_{\mathrm{f}} d\sqrt{\pi^5\nu^3\eta_H\rho}.$$

Then, expression (17) in [20] can be rewritten as follows:

$$\pi^3\nu^2\rho b d^2 + 2bd\sqrt{\pi^5\nu^3\eta_H\rho} = \mu_0\frac{\pi d^2}{2}\left(M_x\frac{\partial H_x}{\partial z}\right)_{z=b/2}. \tag{17}$$

Expression (17) coincides with expression (12) derived above. After elementary algebraic operations with Eq. (17), it is easy to derive a formula for the calculation of the viscosity in the near-wall fluid layer as

$$\eta_H = \frac{1}{\nu^3}\left[\frac{\mu_0 d M_x}{4b\pi\sqrt{\pi\rho}}\left(\frac{\partial H_x}{\partial z}\right)_{z=b/2} - \frac{d^2\nu^2\sqrt{\pi\rho}}{2d}\right]^2. \tag{18}$$

3. Results of calculations. Figs. 1 and 2 show graphically linear approximations of the dependences of the magnetization M for the MF-1 and MF-2 fluid samples on H^{-1} in strong fields, taken from [12]. The experimental data on M was obtained for a magnetic field of $\leq$750 kA/m. The circles in the figures indicate the magnetization values corresponding to the reciprocal value of the magnetic field

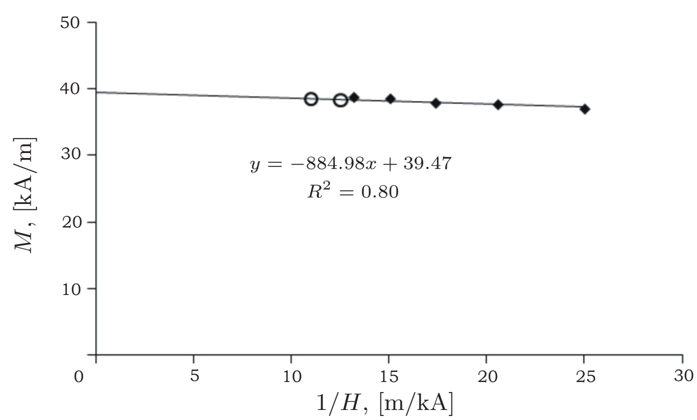

Fig. 1. Dependences $M(H^{-1})$ for the MF-1 sample.

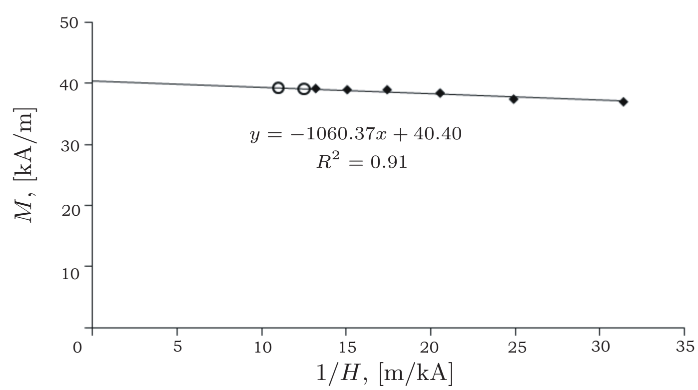

Fig. 2. Dependences $M(H^{-1})$ for the MF-2 sample.

Table 1. Applied combinations of magnetic field parameters and calculated values.

H_0 [kA/m]	H_* [kA/m]	$\Delta H_x/\Delta z$ MA/m^2	M_x [kA/m]	ν [Hz]	η_H [Pa·s]	$\Delta\eta_H$ [Pa·s]	$\Delta\eta_H/\eta$
800	619	14.2	38.3	14.48	0.048	0.013	0.4
900	686	15.9	38.4	15.34	0.052	0.017	0.5

Table 2. Applied combinations of magnetic field parameters and calculated values.

H_0 [kA/m]	H_* [kA/m]	$\Delta H_x/\Delta z$ MA/m^2	M_x [kA/m]	ν [Hz]	η_H [Pa·s]	$\Delta\eta_H$ [Pa·s]	$\Delta\eta_H/\eta$
800	619	14.2	39.1	14.55	0.069	0.039	1.3
900	686	15.9	39.3	15.49	0.066	0.036	1.2

magnetization, 1/800 m/kA and 1/900 m/kA. The formulas in the figures, analytically reflecting the linear approximation of the dependence under consideration, made it possible to calculate the values of M_x. Taking into account the obtained values of M_x and using formula (18), the viscosity values in the near-wall layer in the transverse magnetic field η_H were calculated.

Tables 1 and 2 summarize the applied combinations of the magnetic field parameters, where H_0 is the magnetic field intensity in the centre between the poles of the electromagnet, H_* is the magnetic field intensity, $\Delta H_x/\Delta z$ is the magnetic field intensity gradient at the MF-column base, ν is the oscillation frequency of the MF column, M_x. The tables also list the calculated values of η_H as well as the viscosity increment in the magnetic field (magnetoviscous' effect) $\Delta\eta_H$ and its relative representation $\Delta\eta_H/\eta$.

The obtained results for η_H represent the peak estimate, since the maximum values of the intensity in the pole gap H_0 were chosen in the calculations of M_x. The outlined above limitation of the applied theories makes is possible to select (prefer) the η_H value which is related to the highest magnetic field intensity 900 kA/m.

The interpretation of the difference obtained for the $\Delta\eta_H/\eta$ values in the studied MF-1 and MF-2 samples needs, in particular, comprehensive data on the peculiarities of their structure, but the published experimental results lack them [12, 20].

Conclusion. In this work we evaluate the viscosity increment (magnetoviscous' effect) in a thin near-wall layer of the column of magnetic fluid which oscillates in the tube under the action of a strong transverse magnetic field. The value of the magnetoviscous effect was calculated by the formula derived from two different theoretical approaches and analyzed in relation to the previously published experimental results of the comprehensive measurement of the oscillation frequency and saturation magnetization of two magnetic fluid samples MF-1 and MF-2.

The calculations did not consider the transient thin layer of the passing viscous wave. It was assumed that the entire MF column was involved in the oscillatory motion.

To correct the estimate of η_H, it will be necessary to expand the applicability of the relations obtained, for example, by increasing the magnetic field intensity gradient, which allows generating a higher frequency of the oscillations of the MF column.

Acknowledgements. The investigation was supported by the Ministry of Education of the Russian Federation (state project No. 3.2751.2017/PP).

References

[1] R.E. Rosensweig. *Ferrohydrodynamics* (Cambridge Monographs on Mechanics and Applied Mathematics, New-York, 1985), 344 p.

[2] D.V. Orlov, Yu. Mikhalev, N. Myshkin *et al.* *Magnetic Fluids in Mechanical Engineering* (Eds. D.V. Orlov, V.V. Podgorkov, Moscow, Mechanical Engineering, 1993), 272 p.

[3] S. Taketomi, S. Tikazumi. *Magnetic Fluids* (Moscow, World publishing House, 1993), 272 p.

[4] S. Odenbach (Ed.). *Colloidal Magnetic Fluids: Basics, Development and Application of Ferrofluids* (Lect. Notes Phys., Berlin, Springer, 2009), 430 p.

[5] V.M. Polunin, A.M. Storozhenko and P.A. Ryapolov. *Mechanics of Liquid Nano- and Microdispersed Magnetic Media* (New York – London: CRC Press, CISP, 2017), 210 p.

[6] Wei Zhang, Jinghui Peng and Songjing Li. Damping force modeling and suppression of self-excited vibration due to magnetic fluids applied in the torque motor of a hydraulic servovalve. *Energies*, vol. 10 (2017), pp. 749-759.

[7] V.M. Polunin, M.L. Boev, Myo Min Than, P.A. Ryapolov. Experimental study of an air cavity held by levitation forces. *Magnetohydrodynamics*, vol. 48 (2012), no. 3, pp. 557–566; https://doi.org/10.22364/mhd.48.3.8.

[8] V.M. Polunin, M.L. Boev., Myo Min tan, G.V. Karpova, L.I. Roslyakova. Elastic properties of a magnetic fluid with an air cavity held by levitation forces. *Acoustic Journal*, vol. 59 (2013), no. 1, pp. 63–69.

[9] M.L. Boev, V.M. Polunin, P.A. Ryapolov, G.V. Karpova, P.A. Prokhorov. Oscillations of a bubble separated from an air cavity under compression caused by magnetic field in a magnetic fluid. *Acoustical Physics*, vol. 60 (2014), no. 1, pp. 29–33.

[10] V.M. Polunin, A.M. Storozhenko, I.A. Shabanova *et al.* Effect of the magnetic field perturbation in a magnetic fluid with pulsating bubbles. *Magnetohydrodynamics*, vol. 50 (2014), no. 4, pp. 431–441; https://doi.org/10.22364/mhd.50.4.10.

[11] V.M. Polunin, P.A. Ryapolov, V.V. Platonov, A.E. Kuzko. Free oscillations of a magnetic fluid in a strong magnetic field. *Acoustic Journal*, vol. 62 (2016), no. 3, pp. 302–307.

[12] V.M. Polunin, P.A. Ryapolov, V.V. Platonov *et al.* Elasticity of magnetic fluid in a strong magnetic field. *Acoustical Physics*, vol. 63 (2017), no. 4, pp. 416–423.

[13] J.P. McTaque. Magnetoviscosity of magnetic colloids. *J. Chem. Phys.*, vol. 52 (1969), no 1, pp. 133–136.

[14] R.E. Rosensweig, R. Kaiser, G. Miskolezy. Viscosity of magnetic fluid in a magnetic field. *J. Colloid and Interface Science*, vol. 29 (1969), no. 4, pp. 680–686.

[15] M.I. Shliomis. Effective viscosity of magnetic suspensions. *JETP*, vol. 6 (12) (1971), pp. 2411–2418.

[16] M.I. Shliomis. Magnetic fluids. *Advances in Physical Sciences*, vol. 112 (1974), no. 3, pp. 427–459.

[17] A.O. Cebers. Models of the magnetization of the colloid of a ferromagnet in a hydrodynamic flow. *Magnetohydrodynamics*, vol. 11 (1975), no. 4, pp. 439445.

[18] M.M. Maiorov. Measurement of the viscosity of a ferrofluid in a magnetic field. *Magnetohydrodynamics.*, vol. 16 (1980), no. 4, pp. 339–344.

[19] V.M. Polunin, P.A. Ryapolov, E.V. Shel'deshova *et al.* Study of viscoelastic parameters of magnetic fluids in an external magnetic field. *Proc. the 18th International Plyos Conference on Nanodispersed Magnetic Fluids*, pp. 245–249.

[20] V.M. Polunin, S.G. Emelianov, P.A. Ryapolov, E.V. Shel'deshova. Shear and oscillations of magnetic fluid column in a strong magnetic field. *Magnetohydrodynamics*, vol. 53 (2017), no. 3, pp. 471–481: https://doi.org/10.22364/mhd.53.3.3.

[21] S.N. Rzhevkin. *Lectures on the Theory of Sound* (Moscow, Moscow State University Publ., 1960), 336 p. (in Russian).

[22] V.M. Polunin, P.A. Ryapolov, K.S. Ryabtsev *et al.* Elasticity of the air cavity in a magnetic fluid when changing the sign of the magnetic field of a ring magnet. *Reports of High Education. Physics*, vol. 61 (2018), no. 7, pp. 147–157 (in Russian).

Received 02.11.2018