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THE PROPAGATION OF LINEAR WAVES IN SPIN QUANTUM MAGNETOPLASMAS

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A theoretical investigation on the propagation of linear waves in spin quantum magnetoplasmas is presented. Based on the quantum magnetohydrodynamic model, including the Bohm potential, the relativistic degeneracy pressure of electrons and the spin magnetization energy caused by the electron 1/2 spin effect and the Maxwell's equation modified by the spin current density, the dispersion equation for spin quantum magnetoplasmas is derived. Solving the dispersion equation in the case of propagation parallel or perpendicular to the background magnetic field, dispersion relations of left-handed wave, right-handed wave, upper hybrid wave, ordinary and extraordinary waves are derived, respectively. Research shows that Langmuir oscillations and upper hybrid oscillations can propagate in cold plasmas due to the Bohm potential and relativistic degeneracy pressure. Since the extraordinary wave consists of partial transverse and longitudinal waves, quantum effects can modify its dispersion relation. The modification of the dispersion relations by quantum effects is calculated with typical parameters of a dense astrophysical object, such as the pulsar magnetosphere. It is also confirmed that the pulsar magnetosphere is a real physical environment in which quantum effects need to be considered.

Introduction.

Over the past 5–10 years, quantum plasmas have attracted significant interest due to their extensive applications in quantum wells [1], plasmons [2], ultra-small electronic devices [3], astrophysics [4], ultra-cold plasmas [5–7], and high-energy laser systems [8]. In superdense astrophysical objects, the number density of electrons can be as high as $10^{28\sim30}\,\mathrm{cm}^{-3}$. Under such circumstance, the distribution function of electrons changes from Boltzmann to Fermi–Dirac and the classical thermal pressure is replaced by the degeneracy pressure due to the Pauli exclusion principle. When the de Broglie thermal wavelength becomes comparable to the interparticle distance, electron tunneling effects represented by the Bohm potential become important due to the Heisenberg's uncertainty principle. Several studies have been carried out to investigate the dispersion relation of linear waves using hydrodynamic or kinetics models including the Bohm potential and Fermi statistical pressure [9–17].

Since electrons are Fermions, under the action of a strong magnetic field there will appear an electron spin current and a spin force acting on electrons due to the Bohr magnetization. Spin effects can be significant in extreme astrophysical environments, such as the pulsar magnetosphere, where magnetic fields can be as strong as 10^{12-14} Gs. In 2007, a spin magnetohydrodynamic (MHD) model was proposed by Brodin and Marklund [18], in which electrons were treated as a single fluid. Andreev then gave a generalized form of the quantum hydrodynamic (QHD) model for spin-1/2 particles [19], in which the spin-up and the spin-down electrons were treated as two different fluids. This model is called the separate spin evolution quantum hydrodynamic (SSE–QHD) model. Since then, a lot of studies on the electron spin 1/2 effect have been carried out. Iqbal investigated the spin magnetoacoustic wave and hybrid wave instabilities, which indicated that the dispersion of the upper hybrid wave is affected by the spin effects [20–23]. The extraordinary wave in spin-1/2 quantum plasma was studied by Andreev [24]. Magnetohydrodynamic waves with relativistic electrons and positrons in degenerate spin-1/2 astrophysical plasmas were investigated by Maroof [25]. The magnetohydrodynamic spin wave in degenerate electron-positron plasmas was analyzed by Mushtaq [26].

Using the magnetohydrodynamic model, we investigated the propagation of linear waves in relativistic quantum magnetoplasmas [27] and the dispersion relation of the extraordinary wave and upper hybrid wave in spin quantum magnetoplasmas with a vacuum polarization effect [28]. In this paper, we investigate the propagation of linear waves in dense magnetoplasmas using the quantum magnetohydrodynamic model, in which the Bohm potential, the relativistic degeneracy pressure of electrons, as well as the spin magnetization energy due to the electron-1/2 spin effect are considered. This paper is organized as follows. In Section 1, the dispersion equation for spin quantum magnetoplasmas is derived based on quantum magnetohydrodynamics, including the Bohm potential, relativistic degeneracy pressure and spin force, and the Maxwell equations modified by the spin current. In Section 2, by solving the dispersion equation for the case of propagating parallel or perpendicular to the background magnetic field, the dispersion relations of left-handed wave, right-handed wave, upper hybrid wave, ordinary and extraordinary wave are derived. In Section 3, contributions of the quantum effect are quantitatively calculated with real plasma parameters.

1. Dispersion equation of relativistic quantum plasmas.

Research on the spin effects in quantum plasmas dates back to the works by Marklund and Gordin published in 2007 [16, 18], in which they for the first time present the fully nonlinear governing equations for spin-1/2 quantum electron plasmas. Starting from the Pauli equation for individual particles, they derived one-fluid magnetohydrodynamic equations for different plasmas, including the effects of the electron spin. In this article, we consider zero-temperature isotropic plasmas composed of ions and electrons in a strong magnetic field, in which ions are treated as a stationary neutralizing background, only the movement of electrons is considered. Meanwhile, the electron-ion collisions and nonlinear spin effects are ignored.

The quantum MHD model is composed of the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0, \tag{1}$$

and the electron momentum equation [18,25,26]

$$mn\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{u} = -en\left(\mathbf{E} + \frac{1}{c}\mathbf{u}\times\mathbf{B}\right) - \nabla P + \frac{\hbar^2 n}{2m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{2n\mu_{\mathrm{B}}}{\hbar}\nabla(\mathbf{S}\cdot\mathbf{B}), \tag{2}$$

where $\mu_{\mathrm{B}} = e\hbar/2mc$ is the Bohr magneton, e is the magnitude of the electron charge, m is the electron mass, and $\hbar$ is the Planck constant divided by 2π. The third term on the right side of Eq. (2) is the Bohm potential, and the fourth term on the right side of Eq. (2) is the effective spin force due to the Bohr magnetization. P denotes the relativistic electron degeneracy pressure in dense plasmas which can be written as [8]

$$P = \frac{\pi m_e^4 c^5}{3\hbar^3} f(\xi), \tag{3}$$

where $f(\xi) = \xi(2\xi^2 - 3)(1+\xi^2)^{1/2} + 3\sinh^{-1}(\xi)$, $\xi = p/mc$, and $p = (3\pi^2 n)^{1/3}\hbar$ is the momentum of electron on the Fermi surface. Expanding Eq. (3) around the unperturbed density of electrons n_0 by the Taylor series expansion and neglecting the higher order terms yield [29]

$$P = P_0 + \frac{mv_{\rm Fe}^2}{3\gamma_0} n_1, \tag{4}$$

where n_1 denotes the perturbed electron number density, $v_{\rm Fe} = (3\pi^2 n_0)^{1/3}\hbar/m$ is the Fermi velocity of electron, $\gamma_0 = 1/\sqrt{1-\xi_0^2}$ with $\xi_0 = p_0/mc$, where $p_0 = (3\pi^2 n_0)^{1/3}\hbar$.

By neglecting the spin-thermal coupling terms and the nonlinear spin fluid, the spin vector in Eq. (2) can be written as

$$\mathbf{S} = -\frac{\hbar}{2}\eta\left(\frac{\mu_{\rm B} B}{k_{\rm B} T_{\rm Fe}}\right)\hat{\mathbf{B}}, \tag{5}$$

where B is the magnitude of the magnetic field, $\hat{\mathbf{B}}$ is a unit vector in the magnetic field direction, and $\eta(\alpha) = \tanh\alpha$ is the Brillouin function.

Assuming every quantity in Eq. (2), φ can be written as

$$\psi = \psi_0 + \psi_1, \tag{6}$$

where ψ_0 and ψ_1 are the equilibrium and the perturbation value, respectively. The plasma equilibrium is assumed as $\mathbf{E}_0 = 0$, $\mathbf{u}_0 = 0$, and the linearized continuity equation is derived as

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{u}_1 = 0, \tag{7}$$

and the linearized electron momentum equation is derived as

$$\frac{\partial \mathbf{u}_1}{\partial t} = -\frac{e}{m}\left(\mathbf{E}_1 + \frac{1}{c}\mathbf{u}_1 \times \mathbf{B}_0\right) - \frac{v_{\rm Fe}^2}{3n_0\gamma_0}\nabla n_1 + \frac{\hbar^2}{4m^2 n_0}\nabla\nabla^2 n_1 + \frac{2\mu_{\rm B}}{m\hbar}\nabla(\mathbf{S}\cdot\mathbf{B}_1). \tag{8}$$

The the first order electromagnetic fields $\mathbf{E}_1$ and $\mathbf{B}_1$ in Eq. (8) are governed by the linearized Maxwell equations

$$\nabla \times \mathbf{E}_1 = -\frac{1}{c}\frac{\partial \mathbf{B}_1}{\partial t}, \tag{9}$$

$$\nabla \times \mathbf{B}_1 = \frac{1}{c}\frac{\partial \mathbf{E}_1}{\partial t} + \frac{4\pi}{c}(\mathbf{J}_e + \mathbf{J}_{\rm M}), \tag{10}$$

where $\mathbf{J}_e = -en_0\mathbf{u}_1$ is the free current density of electrons, and $\mathbf{J}_{\rm M} = -c\nabla \times (2n_0\mu_{\rm B}\mathbf{S}/\hbar)$ is the magnetization spin current.

We assume the external magnetic field as $\mathbf{B}_0 = (0, B_0\cos\theta, B_0\sin\theta)$, where θ is the angle between the wave-vector $\mathbf{k} = k\hat{y}$ and the external magnetic field, as shown in Fig. 1. Under the above assumption, the components of the polarized electromagnetic field satisfy the following relations

$$B_{1x} = \frac{kc}{\omega}E_{1z}, \tag{11}$$

and

$$B_{1z} = -\frac{kc}{\omega}E_{1x}. \tag{12}$$

The spin magnetization current density $\mathbf{J}_{\rm M}$ in Eq. (10) can be calculated as

$$\mathbf{J}_{\rm M} = \frac{ic\mu_{\rm B}\eta(\alpha)n_0 k^2 u_{1y}\sin\theta}{\omega}\hat{x}. \tag{13}$$

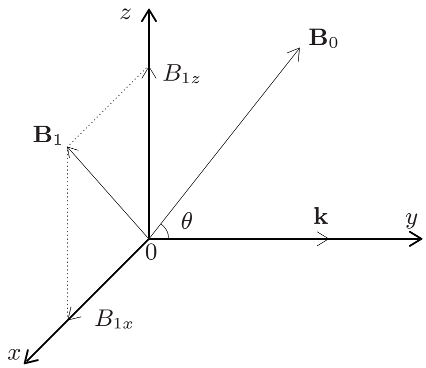

Fig. 1. The Cartesian coordinate system chosen such that $\mathbf{k}$ is along $\hat{y}$, $\mathbf{B}_0$ is in the yoz-plane and $\mathbf{B}_1$ is in the xoz-plane.

Supposing the perturbations are proportional to $\exp[i(ky - \omega t)]$, Eqs. (7) and (8) become

$$n_1 = \frac{ku_{1y}}{\omega} n_0, \tag{14}$$

and

$$\begin{aligned} -i\omega \mathbf{u}_1 = &- \frac{e}{m}\left(\mathbf{E}_1 + \frac{1}{c}\mathbf{u}_1 \times \mathbf{B}_0\right) \\ &- \frac{ikv_{\rm Fe}^2 n_1}{3n_0\gamma_0}\hat{y} - \frac{i\hbar^2 k^3 n_1}{4m^2 n_0}\hat{y} + \frac{i\mu_{\rm B}\eta(\alpha)k^2 cE_{1x}\sin\theta}{m\omega}\hat{y}. \end{aligned} \tag{15}$$

The three components u_{1x}, u_{1y} and u_{1z} of the fluid velocity $\mathbf{u}_1$ can be written as

$$\begin{aligned} u_{1x} &= -\frac{ie}{\omega m}\left(E_{1x} + \frac{u_{1y}}{c}B_0\sin\theta - \frac{u_{1z}}{c}B_0\cos\theta\right), \\ u_{1y} &= -\frac{ie}{\omega m}\left(E_{1y} - \frac{u_{1x}}{c}B_0\sin\theta\right) \\ &\quad + \frac{k^2 v_{\rm Fe}^2 u_{1y}}{3\omega^2\gamma_0} + \frac{\hbar^2 k^4 u_{1y}}{4m^2\omega^2} - \frac{\mu_{\rm B}\eta(\alpha)k^2 cE_{1x}\sin\theta}{m\omega^2}, \\ u_{1z} &= -\frac{ie}{\omega m}\left(E_{1z} + \frac{u_{1x}}{c}B_0\cos\theta\right). \end{aligned} \tag{16}$$

Solving Eqs. (16) yields

$$\begin{aligned} \mathbf{u}_1 = &\frac{e}{m} \\ &\times \begin{vmatrix} \frac{i}{\Omega^2}\left(-\omega + \frac{\omega_{\rm c} S\sin^2\theta}{1-\Delta}\right) & -\frac{\omega_{\rm c}\sin\theta}{\Omega^2(1-\Delta)} & \frac{\omega_{\rm c}\cos\theta}{\Omega^2} \\ \frac{\sin\theta}{1-\Delta}\left[\frac{-S}{\omega} + \frac{\omega_{\rm c}}{\Omega^2} - \frac{\omega_{\rm c}^2 S\sin^2\theta}{\omega\Omega^2(1-\Delta)}\right] & -\frac{i}{\omega(1-\Delta)}\left[1 + \frac{\omega_{\rm c}^2\sin^2\theta}{\Omega^2(1-\Delta)}\right] & \frac{i\omega_{\rm c}^2\sin\theta\cos\theta}{\omega\Omega^2(1-\Delta)} \\ -\frac{\omega_{\rm c}\cos\theta}{\Omega^2} + \frac{\omega_{\rm c}^2 S\sin^2\theta\cos\theta}{\omega\Omega^2(1-\Delta)} & \frac{i\omega_{\rm c}^2\sin\theta\cos\theta}{\omega\Omega^2(1-\Delta)} & -\frac{i}{\omega}\left(1 + \frac{\omega_{\rm c}^2\cos^2\theta}{\Omega^2}\right) \end{vmatrix} \\ &\times \begin{vmatrix} E_{1x} \\ E_{1y} \\ E_{1z} \end{vmatrix}, \end{aligned} \tag{17}$$

where $\omega_{\rm c} = eB_0/mc$ is the electron cyclotron frequency,

$$\Delta = \frac{k^2 v_{\rm Fe}^2}{3\omega^2\gamma_0} + \frac{\hbar^2 k^4}{4m^2\omega^2}$$

is the relativistic quantum correction term,

$$\Omega^2 = \omega^2 - \omega_{\rm c}^2 \cos^2\theta - \frac{\omega_{\rm c}^2}{1-\Delta}\sin^2\theta,$$

and

$$S = \frac{\mu_{\rm B}\eta(\alpha)k^2 c}{e\omega}$$

is the spin correction term.

The dispersion equation for plasmas can be derived from the linearized Maxwell equations as

$$Det\left|\mathbf{kk} - k^2\hat{I} + \frac{\omega^2}{c^2}\hat{\varepsilon}\right| = 0, \tag{18}$$

where $\hat{I}$ denotes the unit tensor, $\hat{\varepsilon} = \hat{I} + \dfrac{4\pi i}{\omega}\hat{\sigma}$ is the dielectric tensor of the medium, and $\hat{\sigma}$ is the conductivity tensor of the medium. The relationship between the current density $\mathbf{J}$ and the conductivity tensor $\hat{\sigma}$ is given by

$$\mathbf{J} = \mathbf{J}_e + \mathbf{J}_{\rm M} = \hat{\sigma}\cdot\mathbf{E}_1. \tag{19}$$

According to Eqs. (17–19), the dispersion equation for spin quantum magnetoplasmas reads as

$$\begin{vmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} \\ \Pi_{31} & \Pi_{32} & \Pi_{33} \end{vmatrix} = 0, \tag{20}$$

where

$$\begin{aligned}
\Pi_{11} =& -k^2c^2 + \omega^2 + \\
&\omega_{\rm p}^2\left\{\frac{-\omega^2}{\Omega^2} + \frac{\omega\omega_{\rm c}S\sin^2\theta}{\Omega^2(1-\Delta)} - \frac{S\sin^2\theta}{1-\Delta}\left[-S + \frac{\omega\omega_{\rm c}}{\Omega^2} + \frac{\omega_{\rm c}^2 S\sin^2\theta}{\Omega^2(1-\Delta)}\right]\right\},\\
\Pi_{12} =& i\omega_{\rm p}^2\left\{\frac{\omega\omega_{\rm c}\sin\theta}{\Omega^2(1-\Delta)} + \frac{S\sin\theta}{1-\Delta}\left[1 + \frac{\omega_{\rm c}^2\sin^2\theta}{\Omega^2(1-\Delta)}\right]\right\},\\
\Pi_{13} =& -i\omega_{\rm p}^2\left[\frac{\omega\omega_{\rm c}\cos\theta}{\Omega^2} + \frac{\omega_{\rm c}^2 S\sin^2\theta\cos\theta}{\Omega^2(1-\Delta)}\right],\\
\Pi_{21} =& -i\omega_{\rm p}^2\frac{\sin\theta}{1-\Delta}\left[-S + \frac{\omega\omega_{\rm c}}{\Omega^2} + \frac{\omega_{\rm c}^2 S\sin^2\theta}{\Omega^2(1-\Delta)}\right],\\
\Pi_{22} =& \omega^2 - \frac{\omega_{\rm p}^2}{(1-\Delta)}\left[1 + \frac{\omega_{\rm c}^2\sin^2\theta}{\Omega^2(1-\Delta)}\right], \quad \Pi_{23} = \frac{\omega_{\rm p}^2\omega_{\rm c}^2\sin\theta\cos\theta}{\Omega^2(1-\Delta)},\\
\Pi_{31} =& i\omega_{\rm p}^2\left[\frac{\omega\omega_{\rm c}\cos\theta}{\Omega^2} - \frac{\omega_{\rm c}^2 S\sin^2\theta\cos\theta}{\Omega^2(1-\Delta)}\right],\\
\Pi_{32} =& \frac{\omega_{\rm p}^2\omega_{\rm c}^2\sin\theta\cos\theta}{\Omega^2(1-\Delta)}, \quad \Pi_{33} = -k^2c^2 + \omega^2 - \omega_{\rm p}^2\left(1 + \frac{\omega_{\rm c}^2\cos^2\theta}{\Omega^2}\right),
\end{aligned} \tag{21}$$

and $\omega_{\rm p}^2 = 4\pi n_0 e^2/m$ is the plasma frequency.

2. Dispersion relation of linear waves.

In this chapter, we discuss the dispersion relation of linear waves in the cases when the direction of propagation is parallel or perpendicular to the background magnetic field.

2.1. Parallel propagation ($\theta = 0$). When the wave vector $\mathbf{k}$ is parallel to the external magnetic field $\mathbf{B}_0$, the dispersion equation Eq. (20) turns out

$$\mathrm{Det}\begin{vmatrix} -k^2c^2+\omega^2\left(1-\frac{\omega_\mathrm{p}^2}{\omega^2-\omega_\mathrm{c}^2}\right) & 0 & e-i\omega_\mathrm{p}^2\frac{\omega_\mathrm{c}\omega}{\omega^2-\omega_\mathrm{c}^2} \\ 0 & \omega^2-\frac{\omega_\mathrm{p}^2}{1-\Delta} & 0 \\ i\omega_\mathrm{p}^2\frac{\omega_\mathrm{c}\omega}{\omega^2-\omega_\mathrm{c}^2} & 0 & -k^2c^2+\omega^2\left(1-\frac{\omega_\mathrm{p}^2}{\omega^2-\omega_\mathrm{c}^2}\right) \end{vmatrix}=0. \tag{22}$$

By solving Eq. (22), we can obtain the dispersion relation for the electron plasma wave propagating parallel to the external magnetic field in spin quantum plasmas as

$$\omega^2=\omega_\mathrm{p}^2+\frac{k^2v_\mathrm{Fe}^2}{3\gamma_0}+\frac{\hbar^2k^4}{4m^2}. \tag{23}$$

It is obvious that the dispersion relation of the electron plasma wave is not affected by the parallel external magnetic field.

The second solution of Eq. (22) is

$$\frac{k^2c^2}{\omega^2}=1-\frac{\omega_\mathrm{p}^2}{\omega(\omega\pm\omega_\mathrm{c})}, \tag{24}$$

which is the dispersion relation of the left-handed wave (L-wave) and the right-handed wave (R-wave) in spin quantum plasmas, respectively.

2.2. Perpendicular propagation ($\theta = \pi/2$). When the wave vector $\mathbf{k}$ is perpendicular to the external magnetic field $\mathbf{B}_0$, the dispersion equation Eq. (20) becomes

$$\mathrm{Det}\begin{vmatrix} -k^2c^2+\omega^2\left[1-\frac{\omega_\mathrm{p}^2(1-\Delta-S^2)}{\omega^2(1-\Delta)-\omega_\mathrm{c}^2}\right] & i\omega_\mathrm{p}^2\frac{\omega_\mathrm{c}\omega+\omega^2S}{\omega^2(1-\Delta)-\omega_\mathrm{c}^2} & 0 \\ -i\omega_\mathrm{p}^2\frac{\omega_\mathrm{c}\omega-\omega^2S}{\omega^2(1-\Delta)-\omega_\mathrm{c}^2} & \omega^2\frac{\omega^2-\tilde{\omega}_h^2}{\omega^2(1-\Delta)-\omega_\mathrm{c}^2} & 0 \\ 0 & 0 & -k^2c^2+\omega^2-\omega_\mathrm{p}^2 \end{vmatrix}=0. \tag{25}$$

The first solution of Eq. (25) is

$$\omega^2=\omega_\mathrm{p}^2+k^2c^2, \tag{26}$$

which is the dispersion relation of the ordinary wave (O-wave) in spin quantum magnetoplasmas.

The second solution of Eq. (25) is

$$\frac{k^2c^2}{\omega^2}=1-\frac{\omega_\mathrm{p}^2}{\omega^2}\frac{\omega^2(1-\Delta-S^2)-\omega_\mathrm{p}^2}{\omega^2-\omega_h^2}, \tag{27}$$

which is the dispersion relation of the extraordinary wave (X-wave) in spin quantum magnetoplasmas, and

$$\omega_h^2=\omega_\mathrm{p}^2+\omega_\mathrm{c}^2+\frac{k^2v_\mathrm{Fe}^2}{3\gamma_0}+\frac{\hbar^2k^4}{4m^2} \tag{28}$$

is the dispersion relation of upper hybrid waves. By setting $\hbar\to 0$ and $S\to 0$, Eqs. (27) and (28) are degenerated to the well-known dispersion relations of the X-wave and upper hybrid oscillation in the classical magnetoplasmas, respectively.

3. Discussion and conclusion.

In this section, we adopted the typical parameters of dense astrophysical objects, such as the pulsar magnetosphere, to a quantitative calculation, where the plasmas parameters were chosen as $n_0 = 10^{29}\,\text{cm}^{-3}$, $B_0 \sim 10^{13}\,\text{Gs}$ and $T \sim 10^9\,\text{K}$ [31]. By choosing such plasma parameters, some basic physical quantities can be calculated as $\omega_\text{p} = 1.78 \times 10^{19}\,\text{s}^{-1}$, $\omega_\text{c} = 1.76 \times 10^{20}\,\text{s}^{-1}$, $v_\text{Fe} = 1.65 \times 10^{10}\,\text{cm/s}$.

It is well known that when the de Broglie wavelength λ_B of electrons becomes comparable to or even larger than the average interparticle distance of electrons (i.e., $\lambda_\text{B}^3 n_0 \sim 1$), the quantum effects play a crucial role in plasma dynamic. From the expression $\lambda_\text{B}^3 n_0 \sim 1$ we calculated the relationship between the number density of electrons and the temperature when the quantum effect should be taken into account as follows

$$\frac{n_0}{T^{3/2}} \sim 10^{16} \frac{\text{cm}^{-3}}{\text{K}^{3/2}}. \tag{29}$$

Obviously, the parameters of the pulsar magnetosphere satisfy the above quantum condition. Therefore, the Bohm potential and Fermi degeneracy pressure should be considered.

The spin effect is important when the energy difference between the two spin states is larger than the thermal or Fermi energy (i.e., $\mu_\text{B} B_0 / K_\text{B} T \geq 1$ or $\mu_\text{B} B_0 / K_\text{B} T_{Fe} \geq 1$) [29]. From the expression $\mu_\text{B} B_0 / K_\text{B} T \geq 1$, we calculated the relationship between the external magnetic field B_0 and the temperature T, when the spin effect should be taken into account, as follows

$$\frac{B_0}{T} \sim 10^4 \frac{\text{Gs}}{\text{K}}. \tag{30}$$

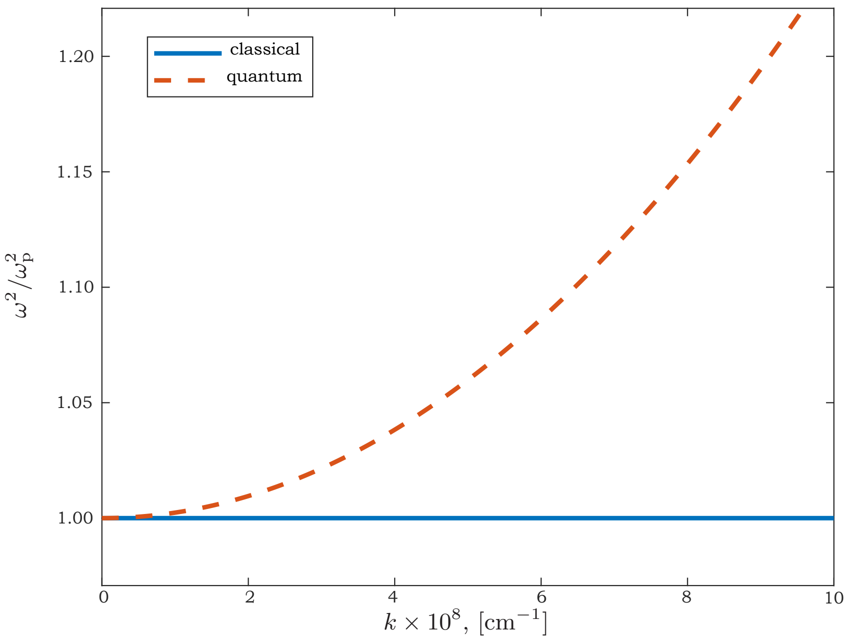

Fig. 2. The Langmuir oscillation (solid line) in classical cold plasmas and the dispersion relation curve (dashed line) in quantum plasmas. The plasma parameters area $n_0 = 10^{29}\,\text{cm}^{-3}$, $B_0 = 10^{13}\,\text{Gs}$.

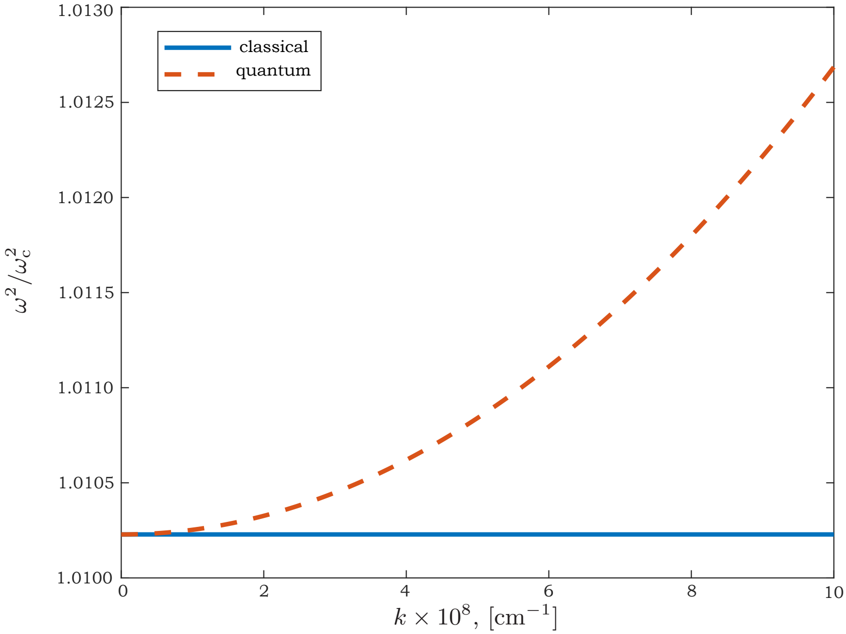

Fig. 3. The upper hybrid oscillation (solid line) in classical cold plasmas, and the dispersion relation curve of the upper hybrid wave (dashed line) in quantum plasmas. The plasma parameters are $n_0 = 10^{29}\,\mathrm{cm}^{-3}$, $B_0 = 10^{13}$ Gs.

From the expression $\mu_B B_0/K_B T_{Fe} \geq 1$, we can also calculate the relationship between the external magnetic field B_0 and the number density of electrons n_0, when the spin effect is taken into account, as follows

$$\frac{B_0}{n_0^{2/3}} \sim 10^{-7}\frac{\mathrm{Gs}}{\mathrm{cm}^{-2}}. \tag{31}$$

Obviously, the parameters of the pulsar magnetosphere meet the above conditions, and the spin effect should be also considered.

Fig. 2 presents the Langmuir oscillation in classical cold plasmas and the dispersion relation of the Langmuir wave in quantum plasmas. Fig. 3 shows the upper hybrid oscillation in classical cold plasmas and the dispersion relation of the upper hybrid wave in quantum plasmas. It was found that the Langmuir oscillation and the upper hybrid oscillation could propagate in cold plasmas due to the Bohm potential and Fermi degeneracy pressure.

In summary, the dispersion relations of linear waves in spin quantum magnetoplasmas were investigated using the quantum magnetohydrodynamic model, including the Bohm potential, Fermi degeneracy pressure and the spin effect. The research has shown that the quantum effects do not affect the propagation of the left-handed wave, right-handed wave and ordinary wave, because they all are transverse waves. It was also found that the upper hybrid oscillation could propagate in cold plasmas due to the Bohm potential and Fermi degeneracy pressure. Since the extraordinary wave consists of partial transverse and longitudinal waves, the quantum effects can modify its dispersion relation. This theoretical study may be useful for comprehending the propagation properties of the high-frequency waves in dense astrophysical objects and also provides an important reference for an experimental study on the intense laser-solid density plasma interaction.

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