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# THE PROPAGATION OF LINEAR WAVES IN QUANTUM MAGNETOPLASMAS WITH VACUUM POLARIZATION EFFECTS

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The vacuum polarization effect may be induced by a strong field and coupled with the plasma process to modify known physical phenomena. In this paper, we present a theoretical investigation on the propagation of linear waves in quantum magnetoplasmas with vacuum polarization effects. Based on the Maxwell equations modified by QED vacuum polarization effects and on the quantum magnetohydrodynamic models in which the Bohm potential and the relativistic degeneracy pressure are taken into account, the dispersion relations for electron plasmas waves, upper hybrid wave, left-handed wave (L wave), right-handed wave (R wave) and extraordinary wave (X wave) in quantum magnetoplasmas are derived, respectively. Theoretical and numerical analysis show that quantum effects and vacuum polarization effect have significant influence on the propagation of linear waves. It is also confirmed that the pulsar magnetosphere is a real physical environment in which quantum effects and vacuum polarization effects need to be considered.

**Introduction.**

Quantum electrodynamics (QED) effects in strong fields is an extensive research field. QED effects has been experimentally confirmed under different conditions, such as the anomalous magnetic moment and the Lamb shift of electrons, but there is still one that has not been verified, called the Schwinger mechanism. In QED theory, it is pointed out that vacuum will exhibit some special properties when a strong field is excitated. For example, when the field intensity reaches the Schwinger's critical strength, the vacuum will be destroyed, and the virtual electron–positron pair can be spontaneously excited into the real electron-positron pair. When the field strength is lower than the Schwinger's critical strength, the vacuum will still show a weak nonlinear dielectric effect due to the quantum fluctuation of the virtual electron–positron pair, which is the so-called QED vacuum polarization effect. The vacuum polarization effect can give rise to many new physical phenomena, such as photon-photon scattering, electron-positron pair generation, vacuum birefringence, photon acceleration in vacuum, and so on.

In the 1930s, there was a theoretical study of the vacuum polarization effect. Delbruck [1] investigated the Delbruck scattering. Heisenberg and Euler [2] as well as Weisskopf [3] obtained the effective Lagrangian of the electromagnetic field, including quantum vacuum correction by the electron-hole theory. It was pointed out that the real part of the effective Lagrangian represented the polarization property of vacuum, whereas the imaginary part corresponded to the generation process of the vacuum electron pair. In 1951, Schwinger used the quantum electrodynamics theory to obtain the same result [4]. Subsequently, researchers carried out related theoretical studies on how strong electromagnetic fields change the dielectric properties of vacuum [5–7].

It is worth noting that the vacuum polarization effect may be induced by a strong field and coupled with the plasma process to modify some known physical phenomena. Meanwhile, the plasma medium may enhance the QED vacuum polarization effect. Piazza found that the vacuum polarization effect can change the refractive index of vacuum, and this change of the refractive index will be amplified in plasma [8]. When a strong laser pulse propagates in a plasma close to the plasma transparency threshold, the vacuum polarization effect is enhanced [9]. Lundin found that the vacuum polarization effect can modify the dispersion relationship of the electromagnetic wave propagation in plasma [10]. Shukla and Stenflo investigated the dispersion relations for elliptically polarized extraordinary waves and linear polarized ordinary waves propagating across an external magnetic field in a dense magnetoplasma [11]. Stenflo investigated a new low-frequency circularly polarized electromagnetic waves in an electron-positron plasma, taking into account the QED effect involving photon-photon scattering [12]. Ji Peiyong studied the correction of photon acceleration in the plasma background by the vacuum polarization effect [13] and the correction of the laser–plasma interaction theory by the vacuum polarization effect [14]. The QED effect can be induced not only by ultra-intense laser, but also by the super strong electromagnetic fields, such as pulsars, whose surface magnetic field intensity is as high as $10^{12}$ Gs, which is close to the Schwinger critical magnetic field. Ji studied the vacuum polarization effects on the electromagnetic radiation propagation in the pulsar magnetosphere [15] and the physical process of the vacuum electron pair induced when the pulsar surface high-energy radiation passes through the plasma layer [16], and obtained the probability of vacuum electron pair generation and discussed the influence of the electron pair generation process on the star surface radiation [17].

Recently, we have studied the dispersion relations of extraordinary waves in quantum magnetoplasmas in which both the spin 1/2 effect and the QED vacuum polarization effect are considered[18], and the dispersion relations of linear waves in quantum magnetoplasmas with the spin 1/2 effect are obtained [19]. In this paper, the propagation of linear waves in quantum plasmas with vacuum polarization effects is investigated by using magnetohydrodynamic (MHD) models in which the quantum effects (Bohm potential and relativistic Fermi degeneracy pressure) and the QED vacuum polarization effects are considered. The paper is organized as follows: in Section 1, the equations of quantum magnetohydrodynamics and the Maxwell's equations modified by the vacuum polarization effect are derived. In Section 2, the dispersion relations of electrostatic waves (electron plasma wave and upper hybrid wave) are obtained by using the equation of quantum magnetohydrodynamics and the Poisson equation. In Section 3, based on the quantum magnetohydrodynamics equation and Maxwell's equation, the dispersion relations of electromagnetic waves (left-handed wave, right-handed wave and extraodinary wave) are derived. In Section 4, we use the real parameters of the pulsar magnetosphere to calculate and discuss the contribution of the quantum effects and QED vacuum polarization effects.

## 1. Basic equations.

In this paper, we consider a quantum plasma composed of ions and relativistic degenerate electrons under the action of a super strong magnetic field. Since the mass of ions is much larger than that of electrons and has no time response to the oscillating field, we can treat ions as a stationary neutralizing background, and only the motion of electrons is considered.

Quantum plasmas can be described by the coupled equations for the density $n$ and fluid velocity $\mathbf{u}$. The density $n$ and the fluid velocity $\mathbf{u}$ satisfy the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0, \tag{1}$$

and the momentum equation

$$mn\left(\frac{\partial}{\partial t}+\mathbf{u}\cdot\nabla\right)\mathbf{u}=-en\left(\mathbf{E}+\frac{\mathbf{u}}{c}\times\mathbf{B}\right)-\nabla P+\frac{\hbar^2 n}{2m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right), \tag{2}$$

where $m$ is the electron mass, $e$ is the magnitude of the electron charge, and $P$ denotes the relativistic electron degeneracy pressure in dense plasmas, which can be written as [20–22]

$$P=P_0+\frac{mv_{\mathrm{Fe}}^2}{3\gamma_0}n_1, \tag{3}$$

where $v_{\mathrm{Fe}}=(3\pi^2 n_0)^{1/3}\hbar/m$ is the Fermi velocity of the electron, $\gamma_0=1/\sqrt{1-\xi_0^2}$ with $\xi_0=p_0/mc$, where $p_0=(3\pi^2 n_0)^{1/3}\hbar$. $n_0$ and $n_1$ denote the equilibrium and perturbation electron number density, respectively.

The QED vacuum polarization effect is expressed by the Heisenberg–Euler Lagrangian density of the electromagnetic field as [23, 24]

$$\mathscr{L}=\frac{1}{8\pi}(E^2-B^2)+\frac{\xi}{8\pi}[(E^2-B^2)+(\mathbf{E}\cdot\mathbf{B})^2], \tag{4}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and the magnetic field, respectively, $E^2$ and $B^2$ are the constants, $\xi=\alpha/45\pi E_{\mathrm{cr}}^2$, $\alpha=e^2/\hbar c$ is the fine structure constant, $B_{\mathrm{cr}}=m^2c^3/e\hbar$ is the critical magnetic field, $\hbar$ is the Plank's constant divided by $2\pi$, $c$ is the speed of light in vacuum. Using the Lagrangian density, the vectors of effective polarization and magnetization of vacuum can be derived as [25, 26]

$$\mathbf{P}=\frac{\xi}{4\pi}[2(E^2-B^2)\mathbf{E}+7(\mathbf{E}\cdot\mathbf{B})\mathbf{B}], \tag{5}$$

and

$$\mathbf{M}=\frac{\xi}{4\pi}[-2(E^2-B^2)\mathbf{B}+7(\mathbf{E}\cdot\mathbf{B})\mathbf{E}]. \tag{6}$$

Assuming every physical quantity, $\varphi$ in Eqs. (1) and (2) can be written as $\varphi=\varphi_0+\varphi_1$, where $\varphi_0$ is the quantity at equilibrium, and $\varphi_1\ll\varphi_0$ is a very small perturbation. The plasma equilibrium is assumed as $\mathbf{E}_0=0$, $\mathbf{u}_0=0$. Under the above assumptions, the linearized continuity equation and the momentum equation can be derived as

$$\frac{\partial n_1}{\partial t}+n_0\nabla\mathbf{u}_1=0, \tag{7}$$

and

$$\frac{\partial\mathbf{u}_1}{\partial t}=-\frac{e}{m}\left(\mathbf{E}_1+\frac{\mathbf{u}_1}{c}\times\mathbf{B}_0\right)-\frac{v_{\mathrm{Fe}}^2}{3n_0\gamma_0}\nabla n_1+\frac{\hbar^2}{4m^2n_0}\nabla\nabla^2 n_1. \tag{8}$$

The linearized Maxwell equations modified by the vacuum polarization effects read as

$$\nabla\times\mathbf{E}_1=-\frac{1}{c}\frac{\partial\mathbf{B}_1}{\partial t}, \tag{9}$$

$$\nabla\times\mathbf{B}_1=\frac{1}{c}\frac{\partial\mathbf{E}_1}{\partial t}+\frac{4\pi}{c}(\mathbf{J}_e+\mathbf{J}_{\mathrm{vac}}), \tag{10}$$

$$\nabla\cdot\mathbf{E}_1=4\pi(\rho_e+\rho_{\mathrm{vac}}), \tag{11}$$

where $\rho_e=-en_1$ and $\mathbf{J}_e=-en_0\mathbf{u}_1$ are the charge and the current density of electrons, respectively. $\rho_{\mathrm{vac}}=-\nabla\cdot\mathbf{P}$ and $\mathbf{J}_{\mathrm{vac}}=\partial\mathbf{P}/\partial\mathbf{t}+c\nabla\times\mathbf{M}$ are the effective vacuum charge and the current density, respectively.

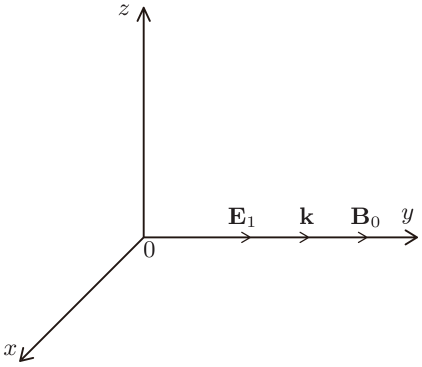


*Fig. 1.* The Cartesian coordinate system chosen such that $\mathbf{B}_0$, $\mathbf{E}_1$ and $\mathbf{k}$ are along $\hat{y}$.

**2. Dispersion relation of electrostatic waves.**

In this section, we assume that the propagation direction of the wave is $\mathbf{k} = k\hat{y}$, and the polarized electromagnetic field can be written as $\mathbf{B}_1 = 0$ and $\mathbf{E}_1 = E_1\hat{y}$. The dispersion relationship of electrostatic waves will be discussed in the following two cases: $\mathbf{k} \parallel \mathbf{B}_0$ and $\mathbf{k} \perp \mathbf{B}_0$.

*2.1. Parallel to the external magnetic field($\boldsymbol{k} \parallel \boldsymbol{B}_0$)* . The wave vector and the polarized electric field are in the same direction as the external magnetic field (see Fig. 1). The charge density induced by the vacuum polarization effects can be written as

$$\rho_{\mathrm{vac}} = -\frac{\xi}{2\pi}\left(E_1^2 + \frac{5}{2}B_0^2\right)\nabla \cdot \mathbf{E}_1. \tag{12}$$

Assuming that the perturbations are proportional to $exp[i(ky - \omega t)]$, Eqs. (7), (8) and (11) can be presented as

$$n_1 = \frac{kn_0}{\omega}u_{1y}, \tag{13}$$

$$-i\omega\mathbf{u}_1 = -\frac{e}{m}\left(\mathbf{E}_1 + \frac{\mathbf{u}_1}{c} \times \mathbf{B}_0\right) - \frac{ikv_{\mathrm{Fe}}^2}{3n_0\gamma_0}n_1\hat{y} - \frac{i\hbar^2k^3n_1}{4m^2n_0}\hat{y}, \tag{14}$$

$$ikE_1 = -4\pi e n_1 - 2ik\xi\left(E_1^2 + \frac{5}{2}B_0^2\right)E_1, \tag{15}$$

By solving Eqs. (13) and (14), the component $u_{1y}$ of the velocity $\mathbf{u}_1$ is obtained as

$$u_{1y} = -\frac{ieE_1}{m\omega(1 - \triangle)}, \tag{16}$$

where $\Delta = \dfrac{k^2v_{\mathrm{Fe}}^2}{3\omega^2\gamma_0} + \dfrac{\hbar^2k^4}{4m^2\omega^2}$ is the quantum correction term.

Substituting Eqs. (13) and (16) into Eq. (15) yields

$$\omega^2 = \frac{\omega_p^2(1 + \Delta)}{1 + \xi(2E_1^2 + 5B_0^2)}, \tag{17}$$

where $\omega_p = (4\pi e^2 n_0/m)^{1/2}$ is the plasma frequency. Eq. (17) is the dispersion relation for the electron plasma wave in quantum plasmas with vacuum polarization effects. When setting $\hbar \to 0$ and $B_0 \to 0$, Eq. (17) can be degenerated to the dispersion relation of Langmuir oscillations in the classical cold plasmas.

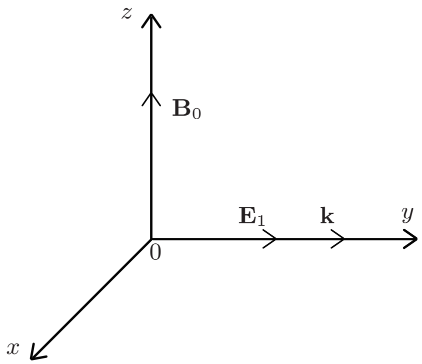


*Fig. 2.* The Cartesian coordinate system chosen such that $\mathbf{B}_0$ is along $\hat{z}$, $\mathbf{E}_1$ and $\mathbf{k}$ are along $\hat{y}$.

*2.2. Perpendicular to the external magnetic field($\mathbf{k} \perp \mathbf{B}_0$)* . Assuming that the propagation of waves is perpendicular to the external magnetic field which is in the $z-$direction ($\mathbf{k} \perp \mathbf{B}_0$ and $\mathbf{B}_0 = B_0\hat{z}$, as shown in Fig. 2), we have $\mathbf{E} \cdot \mathbf{B} = 0$. The charge density produced by the vacuum polarization effects can be written as

$$\rho_{\text{vac}} = -\frac{\xi(E_1^2 - B_0^2)}{2\pi}\nabla \cdot \mathbf{E}_1. \tag{18}$$

Assuming the perturbations are proportional to $exp[i(ky - \omega t)]$, Eq. (11) can be written as

$$ikE_1 = -4\pi e n_1(1+\beta)^{-1}, \tag{19}$$

where $\beta = 2\xi(E_1^2 - B_0^2)$.

By solving Eqs. (13) and (14) which are still applicable in the case of $\mathbf{k} \perp \mathbf{B}_0$, we obtain the component $u_{1y}$ of the velocity $\mathbf{u}_1$ as

$$u_{1y} = -\frac{ieE_1}{m\omega\left(1 - \dfrac{\omega_c^2}{\omega^2} - \Delta\right)}, \tag{20}$$

where $\omega_c = eB_0/m$ is the cyclotron frequency of electrons. When substituting Eq. (20) into Eq. (19), the dispersion relation of upper hybrid waves is derived as

$$\omega^2 = \left(\frac{\omega_p^2}{1+\beta} + \omega_c^2\right)(1+\Delta). \tag{21}$$

When setting $\hbar \to 0$ and $\beta \to 0$, Eq. (21) can be degenerated to the upper hybrid oscillations in classical magnetoplasmas. Eq. (21) indicates that the upper hybrid oscillations can propagate in cold plasmas due to the quantum effects.

## 3. Dispersion relation of electromagnetic waves.

In this section, we assume that the external magnetic field is $\mathbf{B}_0 = B_0\hat{z}$ and the polarized electric field is $\mathbf{E}_1 = E_{1x}\hat{x} + E_{1y}\hat{y}$. The dispersion relationship of electromagnetic waves will be discussed for the following two cases: $\mathbf{k} \parallel \mathbf{B}_0$ and $\mathbf{k} \perp \mathbf{B}_0$.

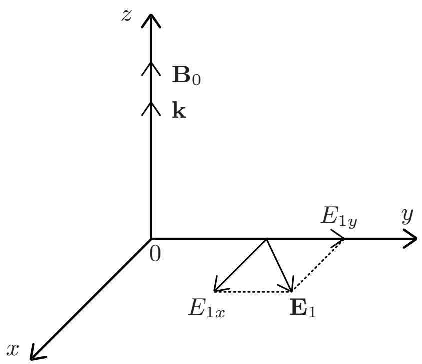


*Fig. 3.* The Cartesian coordinate system chosen such that $\mathbf{B}_0$ and $\mathbf{k}$ are along $\hat{z}$, $\mathbf{E}_1$ is in the $xoy-$plane.

*3.1. Parallel to the external magnetic field($\boldsymbol{k} \parallel \boldsymbol{B}_0$)* . Since the wave propagates along the direction parallel to the external magnetic field ($\mathbf{k} \parallel \mathbf{B}_0$ and $\mathbf{k} = k\hat{z}$, as shown in Fig. 3), the components of the polarized magnetic field can be expressed as

$$\begin{aligned} B_{1x} &= -\frac{ck}{\omega}E_{1y}, \\ B_{1y} &= \frac{ck}{\omega}E_{1x}. \end{aligned} \tag{22}$$

The effective vacuum current density can be written as

$$\mathbf{J}_{\mathrm{vac}} = \frac{\xi}{2\pi}(E_1^2 - B^2)\left(\frac{\partial \mathbf{E}_1}{\partial t} - c\nabla \times \mathbf{B}_1\right). \tag{23}$$

Substituting Eq. (23) into Eq. (10) yields

$$(1+\chi)\left(\nabla \times \mathbf{B}_1 - \frac{1}{c}\frac{\partial \mathbf{E}_1}{\partial t}\right) = -\frac{4\pi e n_0}{c}\mathbf{u}_1, \tag{24}$$

where $\chi = 2\xi(E_1^2 - B^2)$.

Since the perturbations are proportional to $exp[i(kz-\omega t)]$, Eqs. (7) and (8) can be expressed as

$$n_1 = \frac{kn_0}{\omega}u_{1z}, \tag{25}$$

and

$$-i\omega\mathbf{u}_1 = -\frac{e}{m}\left(\mathbf{E}_1 + \frac{\mathbf{u}_1}{c} \times \mathbf{B}_0\right) - \frac{ikv_{\mathrm{Fe}}^2}{3n_0\gamma_0}n_1\hat{z} - \frac{i\hbar^2k^3n_1}{4m^2n_0}\hat{z}, \tag{26}$$

Upon solving Eqs. (25) and (26), the components of $\mathbf{u}_1$ are derived as

$$\begin{aligned} u_{1x} &= -\frac{ie}{m\omega}\left(E_{1x} - i\frac{\omega_c}{\omega}E_{1y}\right)\left(1 - \frac{\omega_c^2}{\omega^2}\right)^{-1}, \\ u_{1y} &= -\frac{ie}{m\omega}\left(\frac{i\omega_c}{\omega}E_{1x} + E_{1y}\right)\left(1 - \frac{\omega_c^2}{\omega^2}\right)^{-1}, \end{aligned} \tag{27}$$

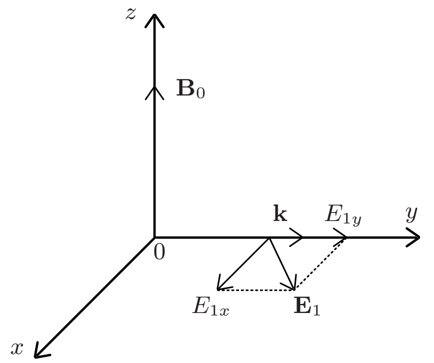


*Fig. 4.* The Cartesian coordinate system chosen such that $\mathbf{B}_0$ is along $\hat{z}$, $\mathbf{k}$ is along $\hat{y}$, $\mathbf{E}_1$ is in the $xoy$-plane.

Based on Eqs. (9) and (24), we derive

$$(\omega^2 - c^2k^2)\mathbf{E}_1 = \frac{4\pi i\omega e n_0}{(1+\chi)}\mathbf{u}_1. \tag{28}$$

Substituting Eq. (27) into Eq. (28) yields

$$\begin{vmatrix} (\omega^2 - c^2k^2)\left(1 - \frac{\omega_c^2}{\omega^2}\right) - \frac{\omega_p^2}{1+\chi} & \frac{i\omega_p^2\omega_c}{(1+\chi)\omega} \\ \frac{i\omega_p^2\omega_c}{(1+\chi)\omega} & -(\omega^2 - c^2k^2)\left(1 - \frac{\omega_c^2}{\omega^2}\right) + \frac{\omega_p^2}{1+\chi} \end{vmatrix} = 0. \tag{29}$$

By solving Eq. (29), the dispersion relation of the left-handed wave (L wave) and right-handed wave (R wave) in quantum-quantum plasmas with the vacuum polarization effects is derived as

$$n^2 = 1 - \frac{\omega_p^2}{(1+\chi)\omega(\omega \pm \omega_c)}. \tag{30}$$

When setting $\chi \to 0$, Eq. (30) can be degenerated to the dispersion relations of L waves and R waves in classical magnetoplasmas.

*3.2. Perpendicular to the external magnetic field($\boldsymbol{k} \perp \boldsymbol{B}_0$)* . Since the wave propagates along the direction perpendicular to the external magnetic field ($\mathbf{k} \perp \mathbf{B}_0$ and $\mathbf{k} = k\hat{y}$, as shown in Fig. 4), the component of the magnetic field perturbations can be expressed as

$$B_{1z} = -\frac{ck}{\omega}E_{1x}. \tag{31}$$

By solving Eqs. (13) and (14), we have

$$\begin{aligned} u_{1x} &= -\frac{ie}{m\omega}\left[(1-\Delta)E_{1x} - \frac{i\omega_c}{\omega}E_{1y}\right]\left(1 - \Delta - \frac{\omega_c^2}{\omega^2}\right)^{-1}, \\ u_{1y} &= -\frac{ie}{m\omega}\left(\frac{i\omega_c}{\omega}E_{1x} + E_{1y}\right)\left(1 - \Delta - \frac{\omega_c^2}{\omega^2}\right)^{-1}, \end{aligned} \tag{32}$$

By solving Eqs. (9) and (24), we derive

$$(\omega^2 - c^2k^2)\mathbf{E}_1 + c^2k^2E_{1y}\hat{y} = \frac{4\pi i\omega e n_0}{1+\chi}\mathbf{u}_1, \tag{33}$$

The component forms of Eq. (33) are

$$\begin{aligned}(\omega^2 - c^2k^2)E_{1x} &= \frac{4\pi i e n_0\omega}{1+\chi}u_{1x},\\ \omega^2E_{1y} &= \frac{4\pi i e n_0\omega}{1+\chi}u_{1y}.\end{aligned} \tag{34}$$

Substituting Eq. (32) into Eq. (34) yields

$$\begin{vmatrix} (\omega^2 - c^2k^2)\left(1-\Delta-\frac{\omega_c^2}{\omega^2}\right) - \frac{\omega_p^2(1-\Delta)}{1+\chi} & \frac{i\omega_p^2\omega_c}{(1+\chi)\omega} \\ \frac{i\omega_p^2\omega_c}{(1+\chi)\omega} & -\omega^2+\omega^2\Delta+\omega_c^2+\frac{\omega_p^2}{1+\chi} \end{vmatrix} = 0. \tag{35}$$

By solving Eq. (35), we obtain the dispersion relation of extraordinary waves (X wave) as

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\frac{\omega^2(1-\Delta)(1+\chi)-\omega_p^2}{(\omega^2-\omega_h^2)(1+\chi)^2}, \tag{36}$$

where

$$\omega_h^2 = \frac{\omega_p^2}{1+\chi} + \omega_c^2 + \frac{k^2v_{\mathrm{Fe}}^2}{3\gamma_0} + \frac{\hbar^2k^4}{4m^2} \tag{37}$$

is the dispersion relation of upper hybrid waves. When setting $\Delta \to 0$ and $\chi \to 0$, Eqs. (36) and (37) can be degenerated to the dispersion relation of extraordinary waves and to the frequency of the upper hybrid oscillations in the classical cold plasmas, respectively.

**4. Discussion and conclusion.**

In this section, we adopt the typical parameters of the dense astrophysical object such as the pulsar magnetosphere for a quantitative calculation, where the plasma parameters are chosen as $n_0 = 10^{29}\mathrm{cm}^{-3}$, $B_0 = 10^{13\sim14}\mathrm{Gs}$, and $T \sim 10^9$ K [27].

The quantum effects play a crucial role in plasmas when the de Broglie wavelength $\lambda_B$ of electrons becomes comparable to or even larger than the average interparticle distance of electrons (viz., $\lambda_B^3 n_0 \sim 1$). From the expression $\lambda_B^3 n_0 \sim 1$, we calculate the relationship between the number density of electrons and the temperature when the quantum effect is taken into account as follows

$$\frac{n_0}{T^{3/2}} \sim 10^{16}\mathrm{cm}^{-3}/\mathrm{K}^{3/2}. \tag{38}$$

Obviously, the parameters of the pulsar magnetosphere satisfy the above quantum condition. Meanwhile, the Fermi velocity of electrons is $v_{\mathrm{Fe}} = 1.65 \times 10^{10}\mathrm{cm/s}$, which is close to the speed of light. Therefore, the Bohm potential and the relativistic degeneracy pressure should be considered.

The QED theory points out that when the field intensity reaches the Schwinger's critical strength, the vacuum will be destroyed and the virtual electron–positron pair can be spontaneously excited into the real electron–positron pair. Therefore, the contribution of the QED vacuum polarization effects to the dispersion relations of linear waves manifests itself in the modification of the plasma frequency. The pulsar surface magnetic field strength is up to $10^{13}$Gs, which is close to the critical magnetic field strength $B_{\rm cr} = 4.4 \times 10^{13}$Gs. The calculation results show that the modification of the plasma frequency by the QED vacuum polarization effects can reach $10^{-5}$, so the QED vacuum polarization effect should be considered.

In summary, we presented a theoretical investigation on the propagation of linear waves in quantum magnetoplasmas with vacuum polarization effects by using the quantum magnetohydrodynamic model. Based on the electron momentum equation containing the Bohm potential, the Fermi degenerate pressure and the Maxwell equations modified by vacuum polarization effects, the dispersion relations of electron plasma waves, upper hybrid waves, left-handed waves, right-handed waves and unusual waves have been derived.

The research showed that the Langmuir oscillation and the upper hybrid oscillation could propagate in cold plasmas due to quantum effects. The dispersion relationships of L and R waves are only affected by the vacuum polarization effect. The quantum effects and the vacuum polarization effects affect the propagation of X waves.

This theoretical study may be useful for comprehending the propagation properties of the high-frequency waves in dense astrophysical objects, and also provide important reference for the experimental study on the intense laser-solid density plasma interaction.

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