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NUMERICAL ANALYSIS OF ANGULAR EFFECT ON LIQUID SILICON FLOW IN THE MHD GRAËTZ HEAT EXCHANGER

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The presented work is a computational study of magnetohydrodynamic flow and heat transfer in the Graëtz heat exchanger with insulating walls in a stable laminar regime under specific temperature conditions at the walls. In all simulations, a numerical method is employed based on finite volume technique. We modelled the process to determine features related to the MHD heat transfer in the Graëtz systems. The combined effect of the magnetic field and plate inclination angle was examined, i.e. under the angular effect on liquid silicon flow, heat and momentum transfer with variable values of the transverse magnetic field strength. The obtained results show that heat transfer is weakly (19%) improved under the applied magnetic field and it greatly (33%) increases under the angular effect. The pressure and velocity fields increase significantly along the axial direction of the modified Graëtz heat exchanger. Such modification has directly affected the distribution of velocity, pressure, and temperature fields, with or without the magnetic field influence. In addition, we can expect a significant stability of the pressure drop along the modified Graëtz channel up to a specified axial position near the downstream region, from which the pressure drop becomes very important for a critical angle $\alpha_c = 9^o$. Thus, MHD heat exchange is improved and the Nusselt number increases monotonically with the increase of the plates inclination angle α.

Key words: Graëtz system, heat transfer, pressure drop, stable laminar regime, magnetic field.

Introduction.

The transfer of heat in a fluid Poiseuille flow when it passes from a hot pipe to a colder pipe of the same diameter is known as the Graëtz problem [1]. The fundamental study of such process began in 1883, its advantage is the possibility to study the evolution of the temperature profile after a sudden temperature change at the walls. In practice, this phenomenon is observed in circular pipes of diameter $D = 2R$, assuming that the flow is permanent and developed, driven by an average velocity U_m. Thus, for $x < 0$, the temperature of the liquid at the entrance region is T_0, and for $x > 0$, the wall temperature is assumed constant T_w (Fig. 1*a*).

The Graëtz heat exchanger subjected to the simultaneous influence of temperature and magnetic fields was studied numerically. A two-dimensional 2D flow of an incompressible electrically conducting viscous liquid silicon (Si) through two types of the Graëtz channel is analyzed. The first Graëtz channel is characterized by a constant cross-section (Fig. 1*b*) and the other one by a modified cross-section (Fig. 1*c*). The efficiency of heat transfer is estimated for the geometric effect and for the flow regime with and without a magnetic field.

In heat transfer techniques, the Graëtz method is an essential control process to influence energy production operations (in nuclear power plants). It is found that a large portion of the thermal energy used in industrial processes is supplied at least once

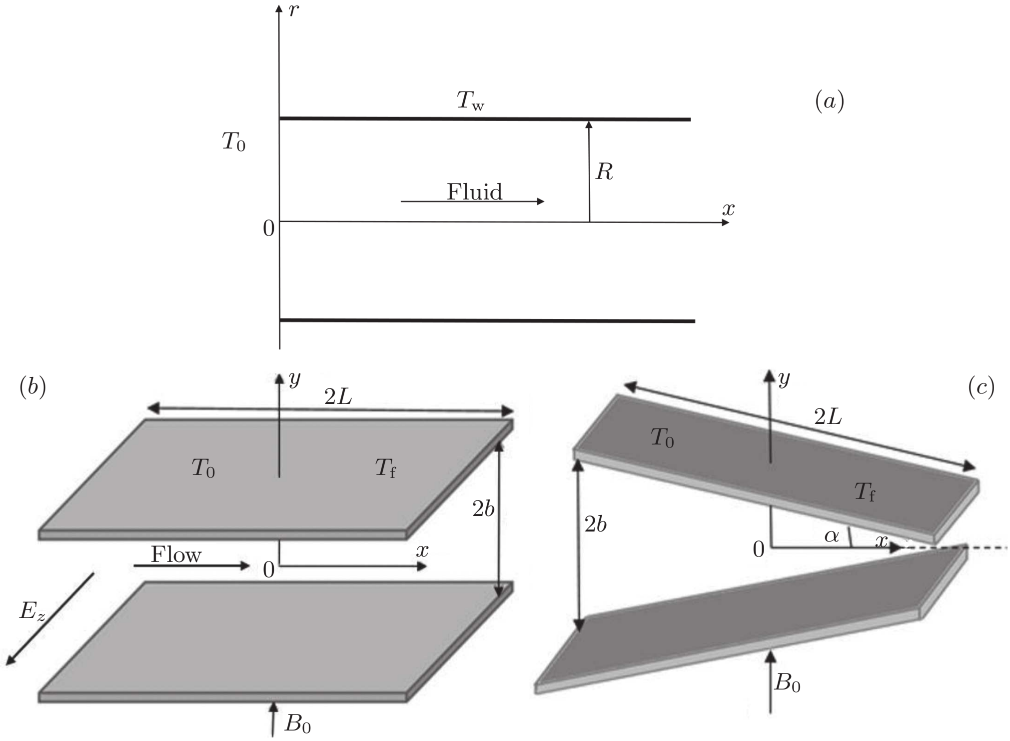

Fig. 1. (*a*). Schematic presentation of the fundamental Graëtz problem. (*b*) The classical Graëtz system, and (*c*) a modified Graëtz system.

through a heat exchanger of this type, as in the processes themselves as in thermal energy recovery systems [2]. This type of process is mainly applied in the production of steel, plastics, and energy. It is well known that a properly sized and pertinent process can lead to very significant productivity gains. We studied a laminar heat transfer through parallel plates with a uniform wall temperature and exposed to a prescribed transverse uniform magnetic field, considering both viscous dissipation and fluid axial heat conduction in the semi-finite thermal entrance region.

With laminar forced convection in a bounded channel when ignoring axial conduction, the heat and mass transfer in cylindrical or parallel plates in a steady state is a well known Graëtz problem [3, 4]. Liou *et al.* [5] applied the method of Papoutsakis *et al.* [4] in heat conduction to solve the problem using a power-law fluid model with a uniform heat flux in a duct and three alternatives for the inlet boundary conditions, including heat generation in the energy equation. They have obtained solutions for temperature profiles and local Nusselt number using the linear operator method. An extended Graëtz problem [6, 7] considers the effect of axial conduction, in particular, in liquids with a low Prandtl number (Pr), such as liquid metals. Problems expanded to two or more contiguous phases by coupling through conjugated conditions of conduction-convection at the boundaries by linking the governing equations in each phase represent the conjugate Graëtz problem [8, 9].

The developed laminar flow of an electrically conducting incompressible fluid or liquid metal through parallel plates exposed to a uniform magnetic field imposed perpendicularly to the walls was studied first by Hartmann [10], then by Hartmann and Lazarus [11] and was expanded by Cowling [12]: this flow was called Hartmann flow. It is well known that the magnetic field influences the heat transfer and hence the temperature distribution by changing the fluid or liquid metal velocity with the Hartmann number (Ha) [10] which is the ratio of electromagnetic forces to viscous forces. Siegel *et al.* [13] analyzed the temperature distribution in the thermal entrance region under the influence of a constant wall heat flux. Nigam *et al.* [14] first investigated the heat transfer in a laminar flow between parallel plates in a duct under the action of a transverse magnetic field in two semi-infinite regions. Michiyoshi *et al.* [15] studied the same problem by neglecting the effects of viscous dissipation and axial heat conduction under a zero-net current open circuit condition. Hwang *et al.* [16] obtained a numerical solution by implicit difference technique, including the effects of viscous and Joule dissipation with a non-zero net current, but still neglecting the effect of axial heat conduction for the case of a uniform wall heat flux condition. Javeri *et al.* [17] investigated the laminar MHD heat transfer coupled with forced convection in the thermal entrance region confined in a parallel plate channel with the third kind boundary conditions by neglecting heat generation. They solved the energy equation by applying the Galerkin-Kantorowich method of variational calculus. Lahjomri *et al.* [18] studied analytically the thermal development of laminar flow through a channel of parallel plates by considering the transverse variation in the fluid temperature, viscous dissipation, Joule heating, and axial heat conduction. Recently, Lahjomri *et al.* [19] applied the same formalism to solve the problem of the Graëtz thermal entrance region in the presence of forced convection of Hagen–Poiseuille flow in a channel with parallel plates considering the heat generation term in the energy equation with Neumann boundary conditions. That study reported that the obtained solution could be applied to any developed flow of liquid metals with heat generation. Many studies have been performed without the magnetic field to solve the thermal heat transfer problem by including or not the axial conduction term in the energy equation [4, 5, 19-27]. Min *et al.* [20], using the method of variables separation, studied a thermal developing flow of a Bingham plastic fluid in a circular pipe with a uniform wall heat flux, and the analytical solution has been presented in terms of the stress yield, Peclet number, and Brinkman number.

Many authors have demonstrated numerically that the MHD action is an effective method to decrease the buoyancy force that results in a natural convective heat transfer in liquid metals through a rectangular cavity [28–30, 35–43]. Afrand *et al.* [30] studied numerically a laminar steady magneto-natural convection in a vertical cylindrical annulus formed by two coaxial cylinders filled with liquid potassium ($\mathrm{Pr} = 0.072$) and exposed to a steady horizontal magnetic field. Their results show that the flow is axisymmetric without the magnetic field, but when exposed to a horizontal magnetic field it becomes asymmetric. This asymmetry is due to the growth of Roberts and Hartmann layers near the walls parallel and normal to the magnetic field, respectively. With a high Prandtl number of $\mathrm{Pr} = 25$ in high Reynolds turbulent channel flows, Yamamoto *et al.* [31] performed a high accuracy DNS with the MHD effect and reported the characteristics of such flow. The obtained data is of considerable importance for quantitative and qualitative investigations of MHD turbulent heat transfer models for the blanket design of fusion reactors. Umeda *et al.* [32] investigated numerically the heat transfer characteristics of a lithium flow in a conducting rectangular channel in the presence of a transverse magnetic field, considering the first wall and blanket cooling for magnetically confined fusion reactors. It is found that the Nusselt number increases by 42–50% with a Hartmann number of 1900, if compared to the hydrodynamic flow, and confirmed that the increase in the Nusselt number with the Hartmann number agrees qualitatively well with the experimental result obtained by Takahashi *et al.* [33].

The main objective of this study is to simulate and evaluate the simultaneous influences of angular and magnetic field effects on the heat transfer in a liquid silicon flow. We are mainly interested in improving the heat transfer efficiency, as well as in better controlling the thermal process that is often encountered in engineering applications.

A new parameter is introduced, i.e. the inclination angle α, which defines the shape of the modified heat exchanger constituting of two converging plates. The Graëtz heat exchanger with parallel plates, named the classical Graëtz heat exchanger, is kept as a reference.

1. Methodology.

In this paper, the studied Graëtz heat exchanger is used in the cooling loop of the nuclear reactor at a nuclear power plant. The study concerns the heat exchanger plates position, which was modified from parallel to inclined by varying the angle α between the plate and the flow direction. Consequently, α is a characteristic parameter that allows a geometrical modification of the Graëtz heat exchanger. If the inclination angle is $\alpha = 0^o$, the system is the classical Graëtz heat exchanger, if the angle is between $0 < \alpha \leq 10^o$, it is called the modified Graëtz heat exchanger. The aspect ratio ($\Gamma = 2L/2b$) was the same for both systems (Fig. 1).

A numerical approach was adopted to solve the equations governing the flow through the considered Graëtz heat exchangers. In these conditions, we performed a simulation using Computational Fluid Dynamics (CFD): FLUENT based on the Finite Volume Method (FVM). In this case, the working conditions describing the Graëtz physical phenomenon were imposed to develop a mathematical model in the considered systems. After that, the associated boundary conditions according to the influence parameters were set up. Therefore, the geometries of the Graëtz heat exchangers are built and meshed under the Gambit software. Then, they were exported to the FLUENT software to solve the governing mathematical model. The phenomenological aspects of the flow and the heat transfer characteristics for the two configurations of the Graëtz heat exchanger in the presence and in the absence of the magnetic field are discussed. The Nusselt number evolution and the pressure drop are determined in the Graëtz MHD channels with electrically insulating walls under specific temperature boundary conditions, taking into account the gravity effect.

2. Mathematical model.

A steady laminar flow of a viscous incompressible and electrically conducting fluid or liquid metal flow with a mean velocity U_{m} in the axial x-direction is considered. It is confined between two parallel plates with the length $2L = 20\,\mathrm{cm}$ separated by the distance $2b = 4\,\mathrm{cm}$ and is exposed to a uniform transverse magnetic field $B^* = B^*_{0y}$ perpendicular to the ox-direction. Here x^*, y^* are the Cartesian coordinates, and the channel is characterized by the aspect ratio $\Gamma = 2L/2b$. It is assumed that the channel walls are electrically insulating ($\chi = -1$) and thermally conducting, maintained at a uniform temperature T_0 for $x < 0$ in the semi-infinite upstream region (entrance region) and at T_{f} for $x > 0$ in the downstream region (Fig. 1). We are interested in the part of the channel, where the flow is fully developed.

The working liquid metal is liquid silicon with constant physical properties at the melting temperature T_{m} (density ρ [kg/m^3], dynamic viscosity μ [kg/ms], electrical conductivity σ [siemens/m], thermal conductivity κ [w/mK] and specific heat C_{p} [J/(kgK)]). The physical properties of liquid silicon and the control parameters are presented in Table 1. Free convection caused by the temperature difference is negligible and the effects of non-zero net current and axial heat conduction are taken into account. Theoretically, for all liquid metals, the Brinkman number is very small, hence, the Joule effect and viscous dissipation can be negligibly small, and the heat transfer is controlled only by convection and conduction. The magnetic diffusivity is constant and equal to the vacuum magnetic diffusivity $\eta = \eta_0$.

Generally, the set of equations which describe MHD for liquid metals flows consists of a combination of the Navier–Stokes equations of fluid dynamics and the equations of electromagnetism, where electromagnetism is characterized mainly by Maxwell equations and Ohms generalized law. These equations are coupled with the energy equation. This MHD system is expressed as

$$\rho\left[\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}\nabla)\mathbf{U}\right] = -\nabla p + \rho\mathbf{g} + \mu\left[\Delta\mathbf{U} + \frac{1}{2}\nabla(\nabla\mathbf{U})\right] + \mathbf{j}\times\mathbf{B}, \tag{1}$$

$$\nabla\cdot\mathbf{U} = 0, \tag{2}$$

$$\mathbf{j} = \sigma(-\nabla\phi + \mathbf{U}\times\mathbf{B}), \tag{3}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{4}$$

$$\nabla\times\mathbf{B} = \eta_0\mathbf{j}, \tag{5}$$

$$\nabla\cdot\mathbf{j} = 0, \tag{6}$$

$$\rho C_{\mathrm{p}}\mathbf{U}\nabla T_i = \kappa\nabla^2 T_i + \mu(\nabla\mathbf{U})^2 + \frac{j^2}{\sigma}, \tag{7}$$

where U [m/s] is the mean velocity, p [pascal] is the pressure, t [s] is the time, ϕ is the electrical potential [V], j [A/m^2] is the current density, η_0 [h/m] is the vacuum magnetic

Table 1. Properties and parameters.

Parameter	**Unit**	**Value**
Channel length $2L$	cm	20
Channel width $2b$	cm	4
Melting temperature T_{m}	K	1683
Thermal conductivity κ	W/(mK)	64
Electrical conductivity σ	Ω/m	106
Density ρ	kg/m^3	2420
Specific heat C_{p}	J/(kgK)	1000
Dynamic viscosity μ	kg/(ms)	$7{\cdot}10^{-4}$
Mean velocity U_{m}	cm/s	$1.8{\cdot}10^{-3} - 1.5$
Prandtl number Pr	–	0.01
Reynolds number Re	–	500 – 2000
Peclet Number Pe	–	5.7 – 22.6
Hartmann number Ha	–	0 – 50
Aspect ratio Γ	–	5
Circuit condition χ	–	–1

diffusivity, and B [T] is the applied magnetic field, i is the region identifier of the Graëtz system: $i = 1$ for the upstream region and $i = 2$ for the downstream region.

To characterize the MHD heat transfer in the Graëtz systems, it is necessary to introduce dimensionless numbers, involving viscous forces, electromagnetic forces, axial conduction, and Joule heating. These dimensionless numbers, which serve as control parameters of the flow, are the Reynolds number $\mathrm{Re} = 2\rho U b/\mu$ which plays the same role in fluid mechanics as the Peclet number $\mathrm{Pe} = 3Ub/2\alpha = \mathrm{Re}\,\mathrm{Pr}$ in heat transfer, the Hartmann number $\mathrm{Ha} = 2B_0 b\sqrt{\sigma/\mu}$, the Prandtl number $\mathrm{Pr} = \mu/(\rho\alpha)$, and the Brinkman number $\mathrm{Br} = \mu U^2/\left[\kappa\left(T_0 - T_\mathrm{f}\right)\right]$, with $\alpha = \kappa/\left(\rho C_\mathrm{p}\right)$.

The MHD flow in an electrically insulating channel at $0 \leq \mathrm{Ha} \leq 50$ and the Prandtl number $\mathrm{Pr} = 0.01$, in a steady laminar state with $500 \leq \mathrm{Re} \leq 2000$ corresponding to the range of the Peclet number $5.7 \leq \mathrm{Pe} \leq 22.6$ is studied. The associated boundary conditions are as follows:

– a fully developed laminar profile at the channel inlet is imposed;

– an outflow, which corresponds to the channel outlet of liquid silicon;

– no-slip boundary condition for the velocity at the walls;

– fixed temperatures, upstream region $T_0 = 2000\,\mathrm{K}$, downstream region $T_\mathrm{f} = 1889\,\mathrm{K}$, the melting temperature $T_\mathrm{m} = 1683\,\mathrm{K}$, and the temperature at the inlet $T_\mathrm{inlet} = 1686\,\mathrm{K}$;

– the thickness of the walls constituting the channel is neglected, it is assumed that the walls are electrically insulating, with the open circuit condition $\chi = -1$;

– the set system of equations and the associated boundary conditions constitute the mathematical formulation of the physical problem under study, considering the assumptions mentioned above.

The set conservation equations is solved by a numerical method. It allows the transformation of partial differential equations into algebraic equations which are simpler to solve.

In this numerical computation, we used the finite volume method to discretize and solve numerically the governing equations with a 2D double-precision solver, the second-order upwind discretization for the energy equation, the SIMPLE algorithm [34] for pressure–velocity coupling, the second-order scheme for pressure interpolation and the second-order upwind scheme for the momentum.

The magnetic equations governing the electric behavior of the conducting liquid metal under the influence of the magnetic induction field are solved using the first-order upwind scheme.

3. Results and discussion.

We used the Gambit software to build the geometry, to generate the meshing and to define the boundary conditions appropriate for the considered system, i.e. interfaces, stiff walls, fluid, etc.

In our investigation, we made a test of adequate meshing using several trials to determine the appropriate number of nodes and to optimize the calculation time and the space of storage before starting the calculation. Therefore, we used a structured quadrilateral meshing consisting of 20480 cells, 41344 faces, and 20865 nodes both for the classical Graëtz heat exchanger and for the modified system, whatever the inclination angle α, with the fixed aspect ratio $\Gamma = 5$ (Fig. 2).

CFD simulations were performed for $\mathrm{Re} = 500$, 1000, and 2000 in the classical and modified Graëtz heat exchangers with different inclination angles α, in the presence or in the absence of the magnetic field. The transverse magnetic field was applied in the perpendicular *oy*-direction of the flow. The investigated Hartmann number values

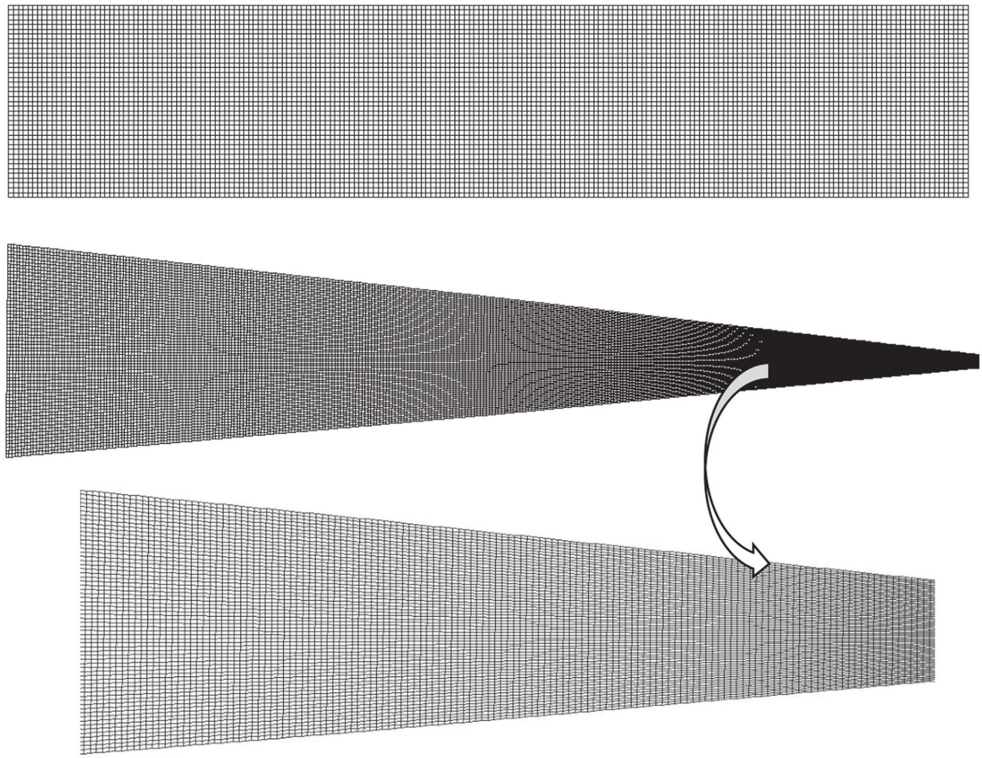

Fig. 2. Meshing of the domain according to different angles α.

were 0, 5, and 50 which correspond to weak intensities of the transverse magnetic field $B_0 = 0$, 0.003 and 0.03 T. Theoretically, liquid metals are characterized by a very small Brinkman number. In the case with liquid silicon under study, the calculation of the Brinkman number has yielded $\mathrm{Br} = 10^{-7}U^2$. This means that, for liquid metals, the effects of viscous dissipation and the Joule effect can be neglected regardless of the value of the magnetic field Ha and flow velocity U for electrically insulating walls $\chi = -1$.

In all considered cases, the pressure, temperature and outlet axial velocity distributions in both Graëtz channels were evaluated. In addition, the Nusselt number profiles were systematically presented in terms of the studied effect. We were interested in the evolution of the local Nusselt number that describes the rate of heat exchange by comparing the conduction mode and the convection mode. The local Nusselt number was analyzed according to the sensitive parameters that affected the Graëtz heat transfer in the classical and in the modified systems.

For a given Reynolds number $\mathrm{Re} = 2000$, the considered liquid silicon flow in the Graëtz heat exchangers was assumed laminar. The Reynolds number was maintained constant and corresponded to a velocity value of 1.5 cm/s. Simulations were made for the Graëtz heat exchanger with different inclination angles α without the magnetic field ($\mathrm{Ha} = 0$).

Fig. 3 illustrates the outlet velocity evolutions along the vertical y-direction in the Graëtz channel for different inclination angles α at $\mathrm{Re} = 2000$, $\Gamma = 5$ without the magnetic field $\mathrm{Ha} = 0$. These profiles are plotted at the outlet of the downstream region of the Graëtz channel for $\alpha = 0^o$, 3^o, 7^o, and 10^o. The flattening of the plotted velocity curves

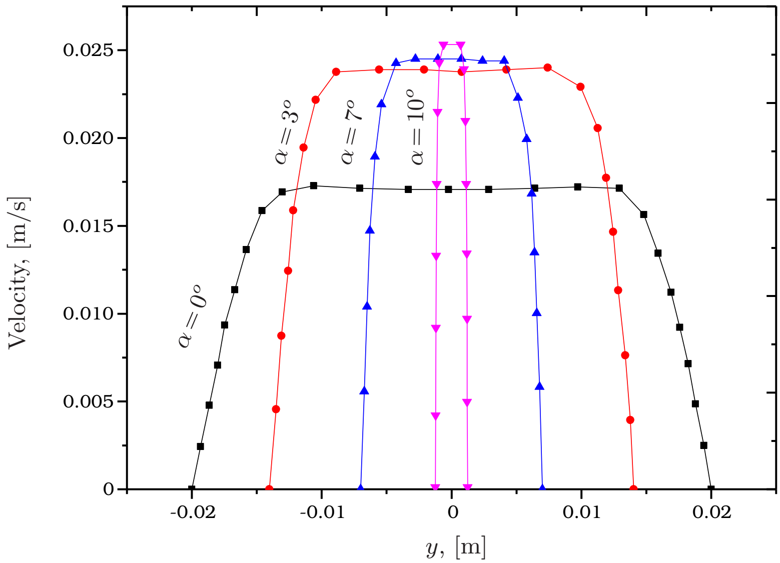

Fig. 3. Outlet velocity evolution along the vertical y-direction in the Graëtz channel for different inclination angles α at $\mathrm{Re} = 2000$, $\Gamma = 5$ without the magnetic field $\mathrm{Ha} = 0$.

with increasing plate inclination angle α is apparent. The velocity increased away from the walls and reached its maximum value in the middle of the Graëtz channel.

This profile's shape is explained by the no-slip condition of the velocity

$$U - U_{\mathrm{wall}} = \beta \frac{\partial U}{\partial n}$$

near the walls and reached its maximum in the middle of the channels, where the liquid silicon flow was completely developed. Indeed, these profiles show the importance of the plate inclination in improving the hydrodynamic process in the modified Graëtz heat exchanger compared to the classical one.

The temperature distribution is presented in Fig. 4 along the two directions of the Graëtz heat exchangers (for $\alpha = 0^o$ and $\alpha = 10^o$): the axial ox-direction and the vertical y-direction at $\mathrm{Re} = 2000$ and $\mathrm{Ha} = 0$, with the same aspect ratio $\Gamma = 5$. Fig. 4a illustrates the variation of the axial temperature at a fixed vertical positions $y = 0$ m and $y = 0.01$ m in the classical Graëtz system ($\alpha = 0^o$) and in the modified Graëtz system ($\alpha = 10^o$) for the same Re, Ha, and Γ. In this case, the temperature of the upper and lower plates constituting the entrance region was kept $T_0 = 2000\,\mathrm{K}$, whereas the downstream region was maintained at $T_{\mathrm{f}} = 1889\,\mathrm{K}$.

The liquid silicon temperature at the inlet was $T_{\mathrm{inlet}} = 1686\,\mathrm{K}$, then the heat was transferred from the plates of the entrance region to the plates of the upstream region and to the whole liquid metal primarily by conduction.

The axial temperature evolution followed an exponential law and increased weakly in the case of the classical Graëtz system compared with the modified system, where the temperature increased, respectively, by 16% and 17.3% from the middle of the channels ($y = 0\,\mathrm{m}$) to the upper wall at $y = 0.01\,\mathrm{m}$.

The vertical temperature evolution for the fixed x-position in Fig. 4b is characterized by a parabolic behavior with a minimum temperature in the middle ($y/2$) of the channels and reaches its maximum value near the walls.

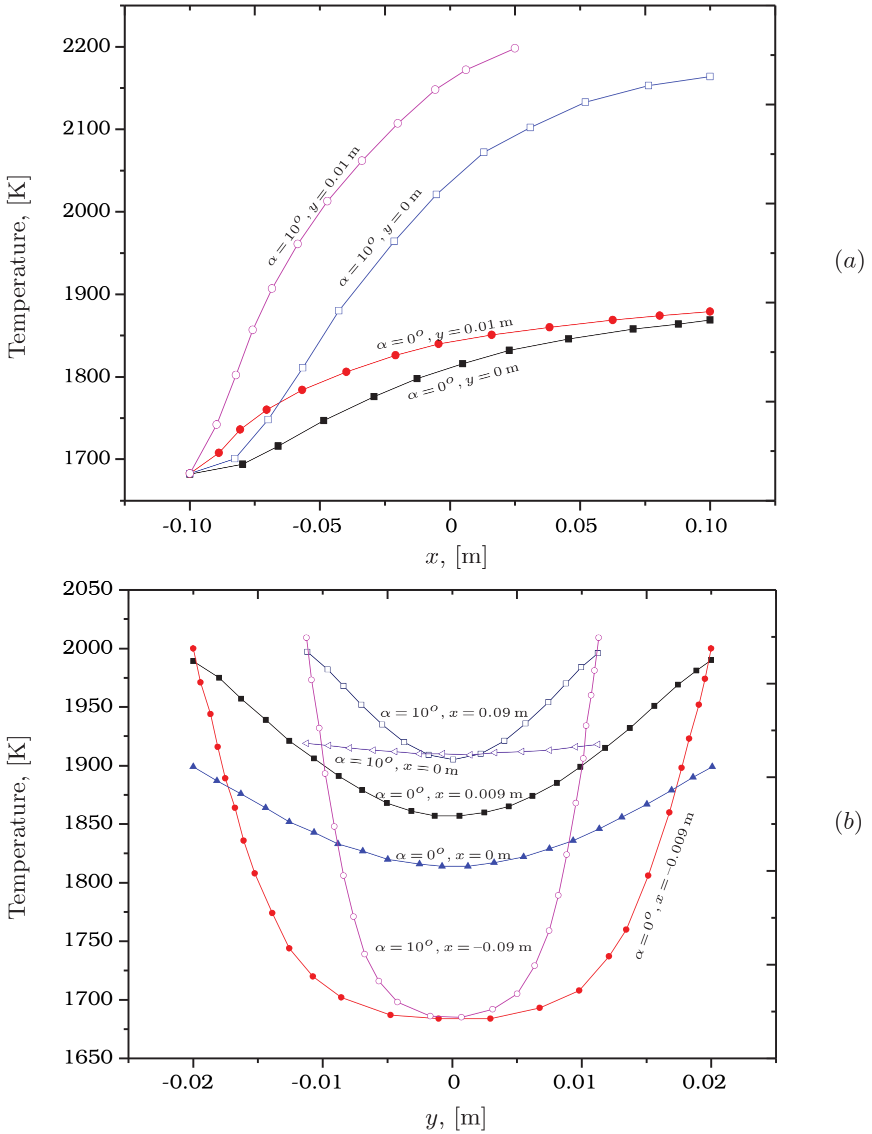

Fig. 4. Temperature profiles of liquid silicon in the Graëtz systems at $\Gamma = 5$, $\mathrm{Re} = 2000$ with no magnetic field ($\mathrm{Ha} = 0$): (a) versus axial x-positions, (b) versus vertical y-positions.

These temperature curves demonstrate enhancement of the heat transfer under the angular effect. The heat transfer increased as the inclination angle increased without the MHD effect ($\mathrm{Ha} = 0$) by 16% along the axial direction for the fixed y-position ($y = 0$ m) and by 19% along the vertical direction for $x = 0.09$ m in the modified Graëtz heat exchange by taking the classical Graëtz heat exchanger as a reference.

To demonstrate the importance of solving the momentum and energy equations without the magnetic field effect, the variation of the mean Nusselt number against the

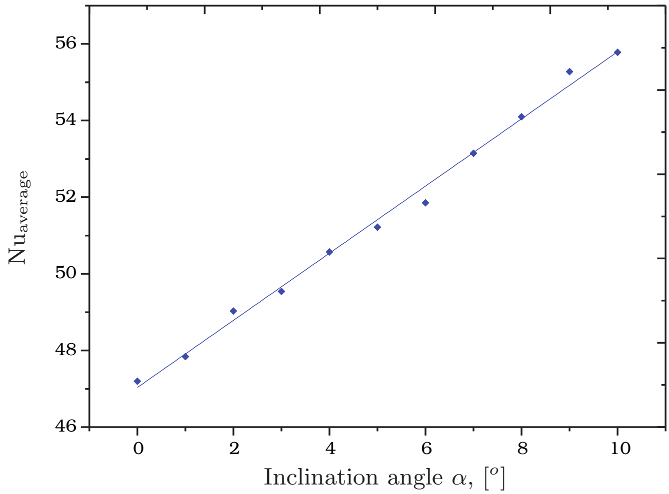

Fig. 5. Mean Nusselt number variation with the inclination angle α at $\Gamma = 5$, $\mathrm{Re} = 2000$ with no magnetic field ($\mathrm{Ha} = 0$).

inclination angle α is illustrated in Fig. 5. As expected, the average Nusselt number increases with increasing inclination angle of the Graëtz channel plates and its evolution is defined by a linear law as shown in Table 2.

The mean Nusselt number for $\mathrm{Re} = 2000$, $\mathrm{Ha} = 0$ with the given set of inclination angles in Fig. 5 ensures the comparison between the studied heat exchangers.

For the case with the modified Graëtz heat exchanger with $\alpha = 10^o$, the mean Nusselt number increases with the increase of the inclination angle, it is larger than in the case with the classical Graëtz heat exchanger with $\alpha = 0^o$, and the enhancement was of about 18.2%.

Fig. 6 depicts the pressure evolution versus the axial positions for different inclination angles at $y = 2\,\mathrm{cm}$, $\Gamma = 5$, without the magnetic field ($\mathrm{Ha} = 0$). It should be emphasized that the pressure field in the entrance region of the classical Graëtz system at $\alpha = 0^o$ decreases quickly until the lower inlet surface extremity of the channel. However, in the modified system, it stabilizes along the entrance region for the abruptly decreasing downstream outlet surface extremity of the flow system with the critical inclination angle $\alpha = \alpha_c = 9^o$. The pressure drop is very important near the outlet of the modified Graëtz heat exchanger with the same inclination angle.

The pressure gradient at $y = 2\,\mathrm{cm}$, $\Gamma = 5$ and $\mathrm{Ha} = 0$ increased considerably along the axial direction of the entrance region as the inclination angle α increased.

Table 2. Mean Nusselt number variation in liquid silicon versus inclination angle α at $\Gamma = 5$, $\mathrm{Re} = 2000$, and $\mathrm{Ha} = 0$.

Equation	$Y = a + bx$ with $Y = \mathrm{Nu}_{\mathrm{average}}$, $x = \alpha$		
		Value	Standard error
Equation characteristics	Intercept, a	47.03409	0.12945
	Slope, b	0.87609	0.02188

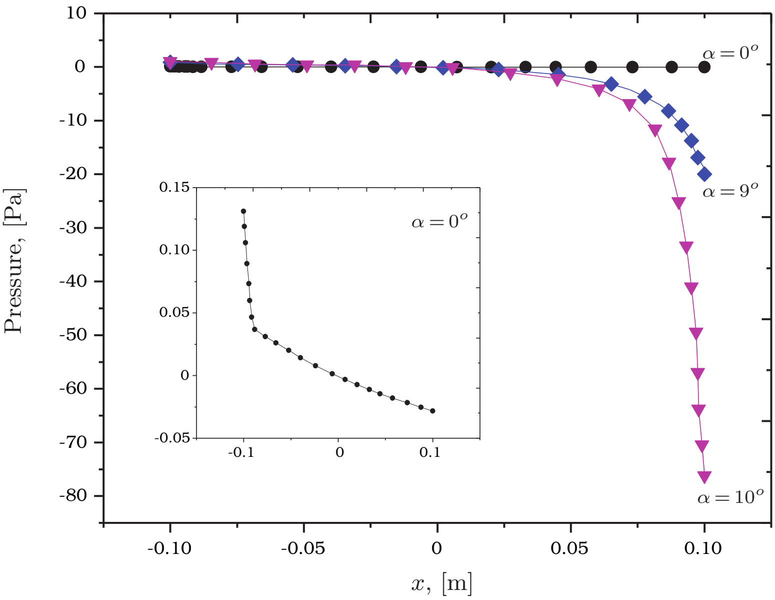

Fig. 6. Pressure evolution versus x-position for different inclination angles α at $y = 2\,\text{cm}$, $\Gamma = 5$, and $\text{Ha} = 0$.

The new Graëtz system seems to demonstrate very interesting features of momentum and heat transfer, its unique properties were obtained even in the absence of the magnetic field $\text{Ha} = 0$. This means that the heat exchange becomes considerable when the inclination angle α of the modified Graëtz system increases.

Free convection was assumed to be negligible, the effect of forced convection due to the flow velocity was examined in both Graëtz heat exchangers for $\text{Re} = 500$, 1000 and 2000. Forced convection plays a decisive role for the heat exchange in the heat exchangers and energy production systems. It is directly related to the momentum transfer characterized by the Reynolds number. The heat transfer fluid was liquid silicon (Si) characterized by the Prandtl number $\text{Pr} = 0.01$ and, since the parameters of the simulation were dimensionless, we tried to accentuate the effect of the Re number variation on the flow in the systems studied previously: in the classical system ($\alpha = 0^o$) and in the modified system ($\alpha \neq 0^o$). The modified Graëtz heat exchanger in this case was studied with $\alpha = 10^o$. The two examined systems were defined by the aspect ratio $\Gamma = 5$.

The pressure field, the temperature distribution, and the Nusselt number are evaluated in the figures by considering the Re number without the effect of the magnetic field in the classical and modified Graëtz heat exchangers.

Fig. 7 illustrates the pressure evolution versus the x-position for different Re numbers at the upper plate of the channel ($y = 2\,\text{cm}$), with $\Gamma = 5$ and $\text{Ha} = 0$ in both Graëtz systems: classical (a) and modified (b). The pressure evolution in Fig. 7a is determined by the presence of two zones: zone with a high-pressure gradient near the system inlet, and zone with a weak pressure gradient along the classical Graëtz channel at $\text{Re} = 500$ and 1000, with the highest pressure gradient in this zone observed at $\text{Re} = 2000$. In the modified Graëtz heat exchanger, the pressure was almost constant along the entrance

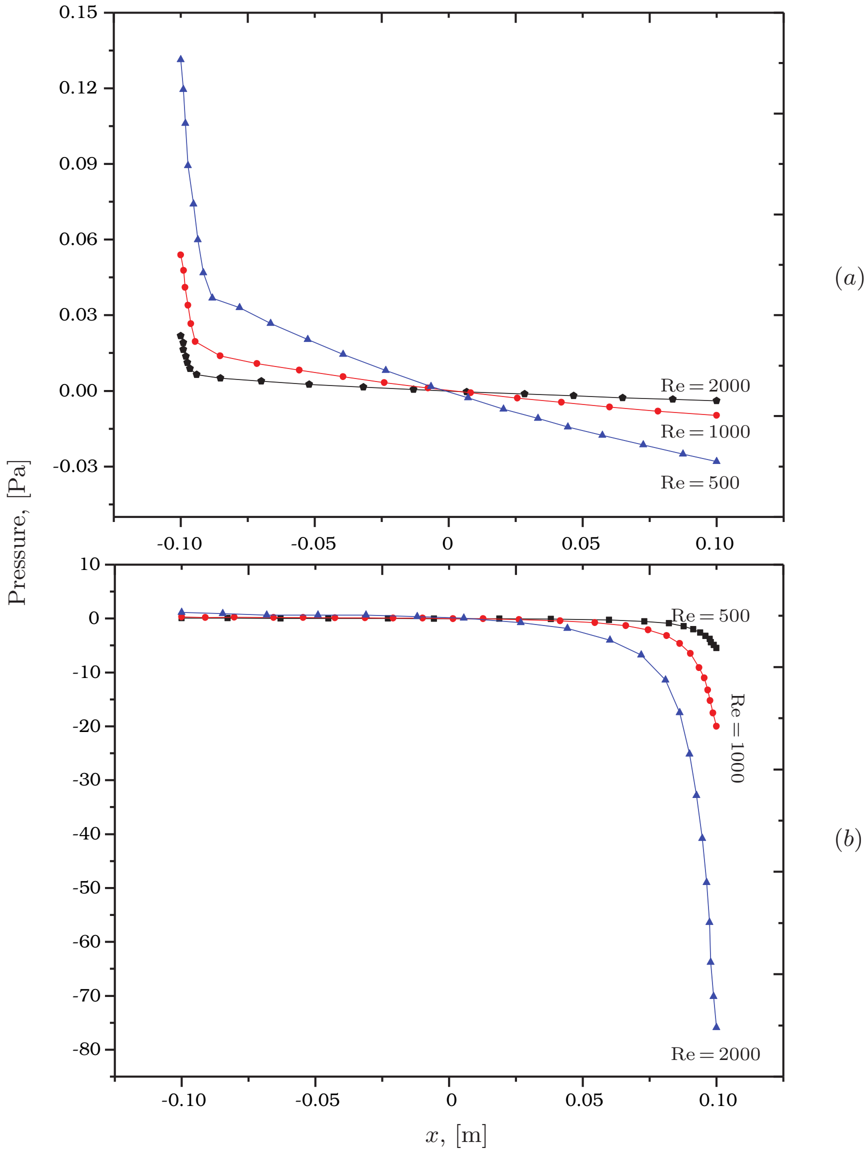

Fig. 7. Pressure evolution versus x-position for different Re numbers at $y = 2$cm, $\Gamma = 5$ and Ha = 0 in the classical Graëtz system (a) and in the modified Graëtz system (b).

and the singularity regions, whatever the value of the Reynolds number, and it suddenly decreased near the downstream region outlet until a high-pressure drop was achieved. When the Re number increased, the pressure increased, and a high-pressure drop was observed at Re = 2000 in the modified Graëtz system (Fig. 7b).

The axial temperature evolution of liquid silicon along the x-direction for different Reynolds numbers at $y = 0$ m and $\Gamma = 5$ in the classical system and in the modified system, without the magnetic field impact, is plotted in Fig. 8. The temperature development

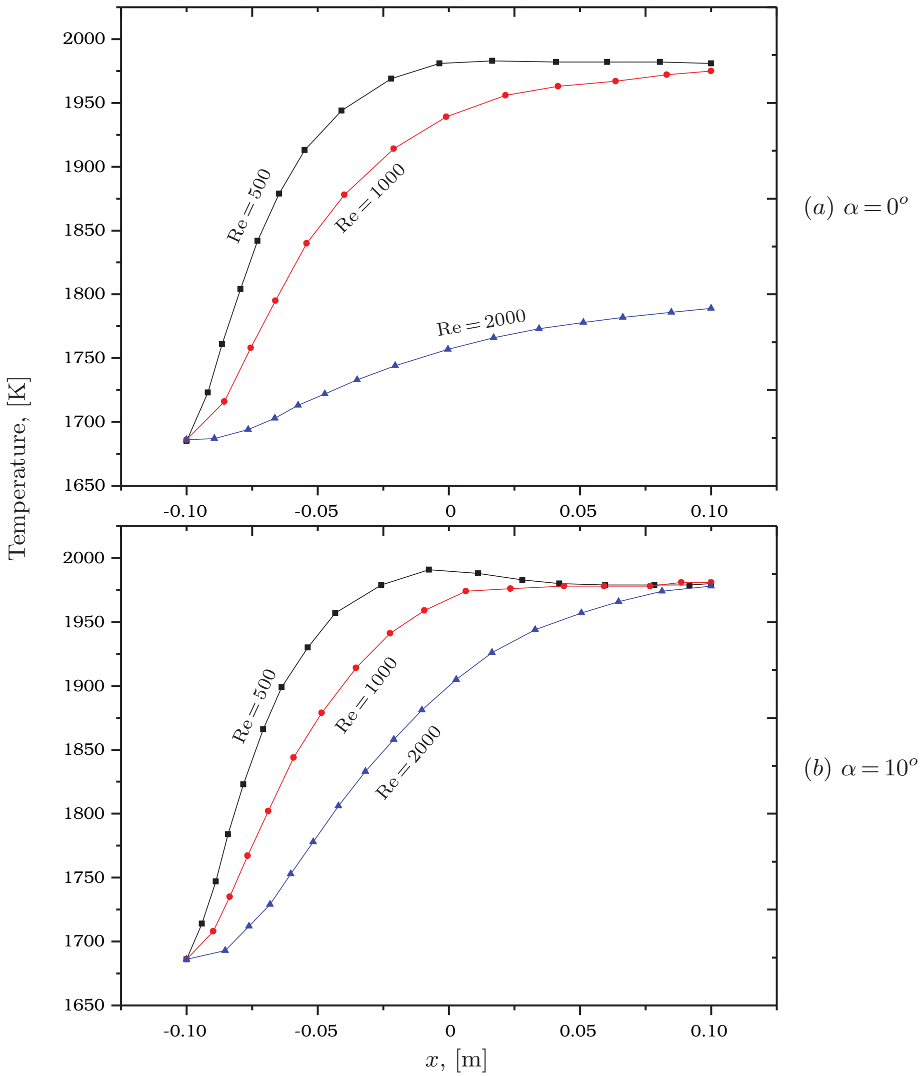

Fig. 8. Axial temperature evolution of liquid silicon along the x-direction for different Re at $y = 0$ m and $\Gamma = 5$ in the classical system (a) and in the modified system (b), without the magnetic field.

along the axial ox-distance in Figs. 8a,b shows that the temperature of the liquid silicon increases along the axial direction for $y = 0$ m and a fixed value of the Re number in both considered Graëtz channels. Fig. 8 shows that the temperature decreases with increasing Reynolds number and the outlet temperature is lower with the high values of Re. It reaches a value of 1978 K at Re = 2000 in the modified Graëtz channel and of 1750 K in the classical Graëtz channel, with the same Re value. Knowing that the studied systems are intended for cooling liquid metals ($T_0 > T_{\mathrm{f}}$), the liquid silicon is cooler in the classical Graëtz heat exchanger than in the modified Graëtz system when the Re number increases.

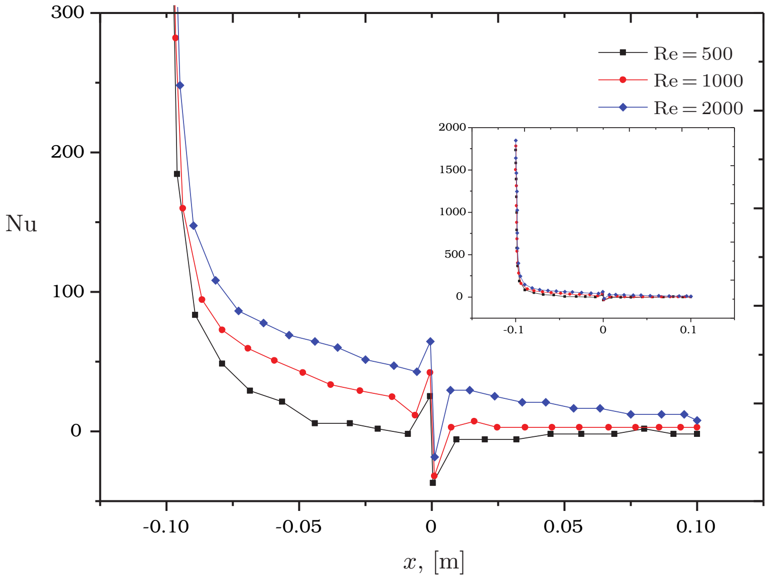

Fig. 9. Nusselt number evolution along the axial direction for different Re, $\Gamma = 5$ and $\mathrm{Ha} = 0$ in the modified system with $\alpha = 10^o$.

Fig. 9 qualitatively illustrates the behavior of the Nusselt number Nu as a function of the axial positions for different Reynolds numbers in the absence of the MHD effect in the modified Graëtz system. The Nu number is generally used as an index of the horizontal heat transfer from the walls to the liquid silicon.

By analyzing the plotted curves in Fig. 9 which characterize the evolution of the Nusselt number defined as the ratio of convective heat transfer to heat conduction, it was observed that Nu was very sensitive to the flow velocity along the axial direction of the Graëtz channel; it varies linearly and proportionally with the Reynolds number. It tends to exponential decreasing along the thermal entrance region and increases under the forced convection effect. In the singularity zone (at $x = 0$ m), where cooling begins, the heat transfer increases considerably at the inlet towards the entrance region and decreases suddenly at $x = 0$. We observed a weak increase of the Nusselt number from the singularity zone towards the downstream region inlet, where the thermal equilibrium was set up in the vicinity of the channel outlet. Hence, the increase of the Re number is proportional to the increase of the convective heat transfer.

Maximum and minimum values of the Nusselt number describing the entrance region corresponding to the studied channels in terms of the Reynolds number are summarized in Table 3.

Table 3. Re number versus Nu number.

	Classical system ($\alpha = 0^o$)		Modified system (α=10^o)	
Re	$\mathrm{Nu}_{\mathrm{max}}$	$\mathrm{Nu}_{\mathrm{min}}$	$\mathrm{Nu}_{\mathrm{max}}$	$\mathrm{Nu}_{\mathrm{min}}$
500	1634	37.8	1741	37.1
1000	1677	45.9	1783	44.0
2000	1755	62.8	1858	68.8

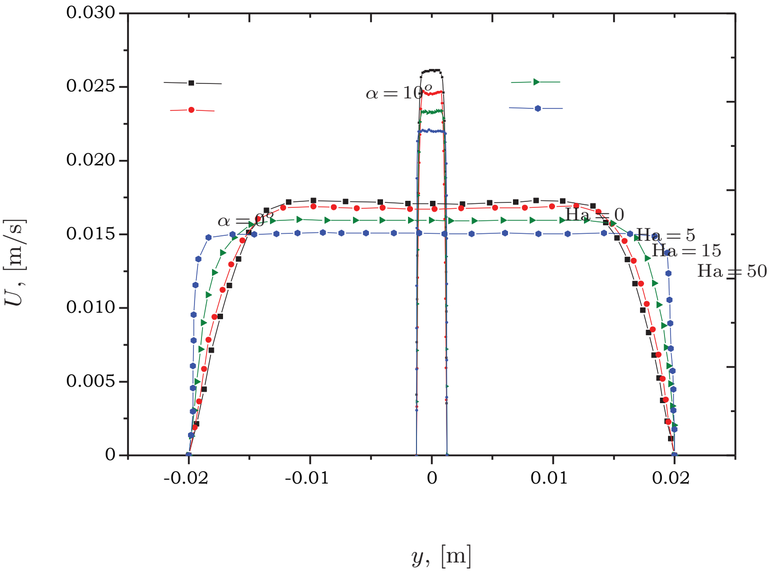

Fig. 10. MHD effect on the outlet velocity along y-positions at Re = 2000 in the classical and modified Graëtz channels.

As to the evolution of the Nusselt number, we can select a system which additionally enhances the heat transfer due to different imposed conditions, such as the shape and the regime. It seems that the modified Graëtz system is more thermally efficient than the classical Graëtz system. The heat exchange was improved under the effect of the inclination angle by approximately 18.2% in the modified Graëtz system ($\alpha = 10^o$) if compare to the classical Graëtz system, with the same Re = 2000. In both Graëtz systems, the heat transfer was increased by 7.4% under the effect of the Reynolds number, varying from Re = 500 to Re = 2000.

In order to evaluate the simultaneous impact of the magnetic field and angular effect on the flow in the Graëtz channels, the Ha number was varied from Ha = 0 to Ha = 50, which yielded directly the value of the applied magnetic field B. Different numerical calculus related to the magnetic field intensity have made it possible to plot the outlet velocity, temperature and pressure evolutions in terms of the x-positions and inclination angle α for $\Gamma = 5$ in the laminar regime.

A comparison of the outlet velocity for both cases of the considered Graëtz problem versus the y-positions at Re = 2000 in MHD channels is presented in Fig. 10. The graphs show that the MHD impact considered in the studied Graëtz systems affects the electrically conducting liquid metal flow (liquid silicon) and generates in it mechanical

Table 4. Ha number versus velocity $U_{\max}$ in the Graëtz systems.

Ha	$U_{\max} \times 10^{-2}$ [m/s]	$U_{\max} \times 10^{-2}$ [m/s]
0	1.726	2.613
5	1.689	2.467
15	1.592	2.339
50	1.513	2.211

and thermal effects, whose intensities depend essentially on the characteristics (intensity and direction) of the applied magnetic field. As the applied magnetic field is a resistive force which decreases the motion of liquid silicon, the velocity field decreases in both Graëtz heat exchangers. It was observed that under the imposed external magnetic field, the velocity was varied, and the flow was completely developed in the singularity and upstream regions (Table 4).

With no magnetic field, the mean flow was maximum in the middle of both Graëtz systems regarding the inlet boundary condition and it was minimum near the upper and lower plates of the systems due to the no-slip boundary condition. For the case with the vertical magnetic field, the velocity profiles became flat in the flow core and decreased strongly near the insulating walls of the Graëtz systems with the increase of the magnetic field intensity.

In Fig. 11, the pressure field distribution seems relatively modified according to the Ha number. This pressure appears much influenced by the high values of the magnetic field because the pressure force and the Lorentz force interact with each other and vary proportionally in the two considered systems.

The singularity region located between the upstream and downstream regions of the Graëtz heat exchanger, where the cooling starts without phase change, was significantly affected by the applied magnetic field in the modified Graëtz system. Fig. 11*b* analyzes the pressure distribution for the MHD problem. It demonstrates that with $\mathrm{Ha} = 0$ the pressure is almost constant along the inlet and singularity regions until high-pressure gradients are achieved near the downstream region. In the case with the magnetic field action, the pressure distribution in both Graëtz systems has the same shape and the pressure gradients are weak under the MHD effect. Fig. 11*a* demonstrates that this gradient became considerably important at $\mathrm{Ha} = 50$.

The presence of the magnetic field influenced weakly the downstream region in the modified Graëtz system: it was observed that the pressure profiles were not sensitive to the MHD effect, and a constant pressure gradient was achieved whatever the Ha value near the outlet of the modified Graëtz system.

The mathematical equations of the curves characterizing the mean Nu number evolution as a function of the Ha number in the two studied Graëtz systems are presented in Table 5 for $\mathrm{Re} = 2000$, $\Gamma = 5$ and $\mathrm{Pr} = 0.01$.

A similar behavior of both studied systems is seen in Fig. 12, which presents the average Nu numbers evaluated for the combined heat transfer and hydromagnetic flow at $\mathrm{Re} = 2000$. The results are presented for different values of the Ha number, considering again the viscous or Joule heating effects and no conducting walls. It is possible to see that with lower Ha values [0, 20] the average Nu number increases quickly, and the slope

Table 5. Average Nu number evolution versus Ha number in classical and modified systems for $\mathrm{Re} = 2000$ and $\Gamma = 5$.

Equation		$Y = a + B_1 x + B_2 x^2$ with $Y = \mathrm{Nu}_{\mathrm{average}}$, $x = \mathrm{Ha}$			
		Classical system ($\alpha = 0^o$)		Modified system ($\alpha = 10^o$)	
		Value	Standard error	Value	Standard error
Equation characteristics	Intercept, a	47.46153	0.8009	56.29878	0.94103
	B_1	0.83965	0.11775	0.96658	0.13835
	B_2	–0.01103	0.00219	–0.01265	0.00257

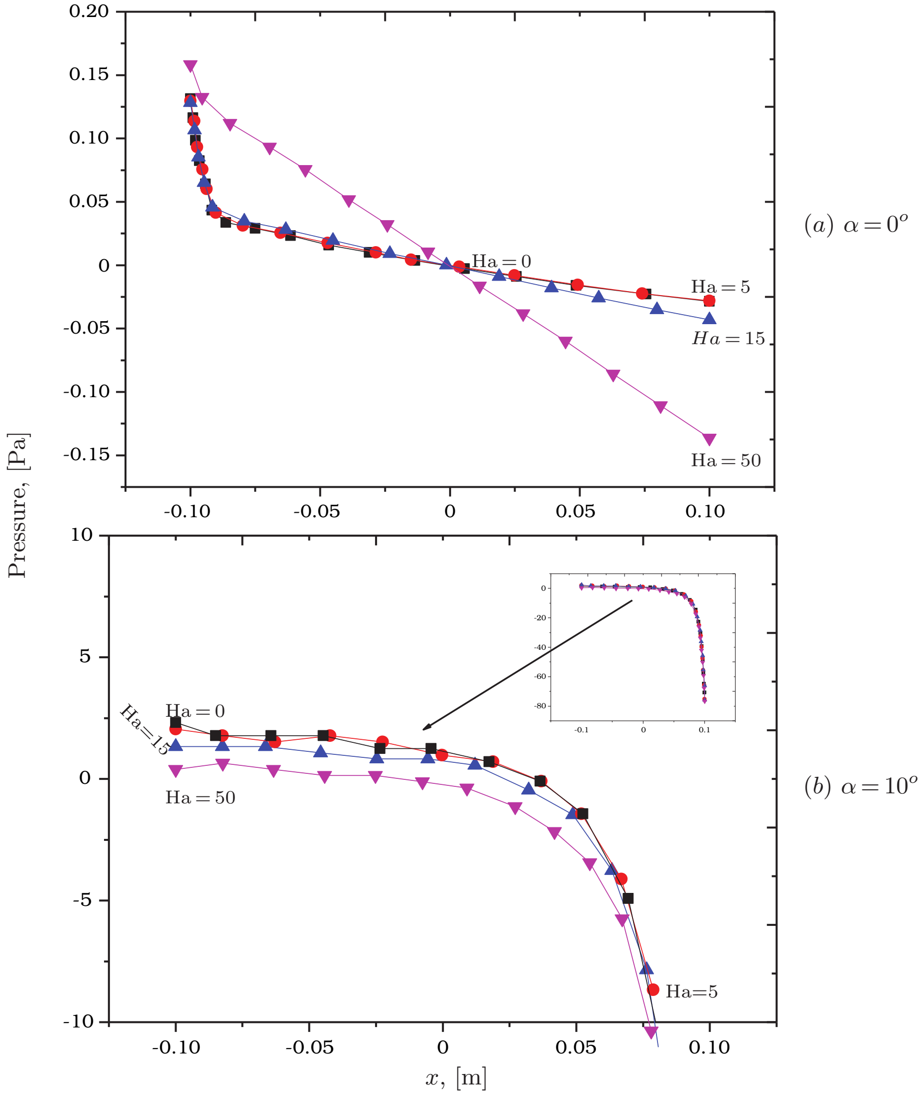

Fig. 11. MHD effect on the pressure field along x-positions at $y = 0.02$ m and Re = 2000 in the classical (a) and modified (*b*) Graëtz channels.

is very pronounced, but at Ha > 20, the Nu number increases insignificantly and reaches its maximum value at Ha = 40.

Fig. 12 indicates that the magnetic field improves the heat transfer in both Graëtz heat exchangers. As the magnetic field intensity increases with Re = 2000, the convective heat transfer is also improving. The heat transfer is more important in the modified Graëtz heat exchanger and increases by 19% if compare with the classical Graëtz heat exchanger with a constant magnetic field.

In the modified system, the MHD effect improved the heat transfer through the liquid silicon by 33% at Ha = 0 to Ha = 50.

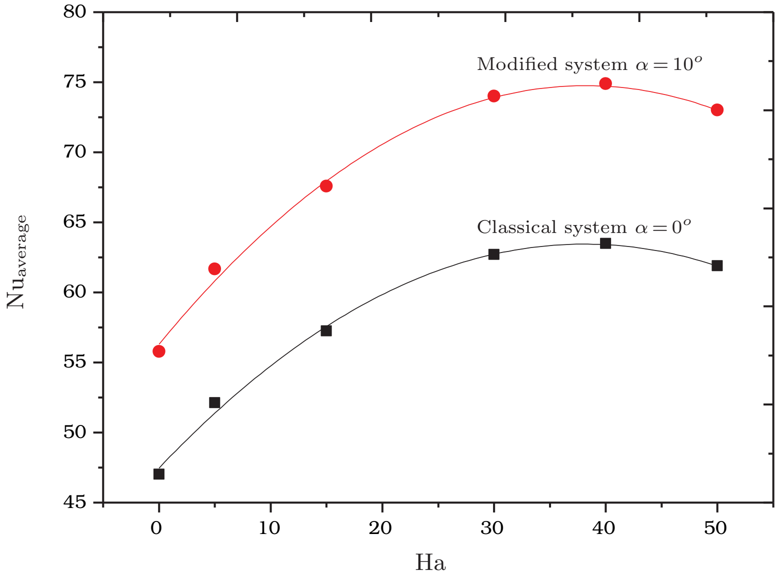

Fig. 12. Average Nu number variation versus Ha number in the classical and modified systems at Re = 2000.

Conclusion.

From the numerical prediction, interesting results were obtained from a simulation with dimensionless parameters using the FLUENT CFD computational code based on the finite volume method. The distributions of temperature, pressure and velocity in the Graëtz flow systems (classical and modified) were determined.

First, the angular effect on the liquid metal flow was studied in terms of the heat transfer rate and efficiency. Second, we deduced the importance of forced convection in improving the thermal properties of the process, particularly, in the modified Graëtz heat exchanger. Then, we emphasized the effect of the magnetic field in the classical and in the modified Graëtz systems.

Therefore, a hydrodynamic chart (pressure field and velocity distribution) was plotted, a systematic and comparative study was performed for both classical and modified Graëtz systems, and it has been revealed that the magnetic field can decrease the intensity of forced convection in the classical system, whereas it maintains in the modified one. Under the working conditions corresponding to Pr = 0.01, Re = 2000 and Ha = 50, the Nusselt number can reach Nu = 414 in the modified Graëtz heat exchanger characterized by $\alpha = 10^o$. According to the obtained results, we determined the most convenient system in terms of efficiency and heat exchange, and it is found that the modified geometry presents unique qualities even in the absence of the magnetic field. The main reason is probably due to the better exchange and control of the entrance region of the Graëtz system. In addition, the modified Graëtz system homogenized the heat transfer and momentum by stabilizing the pressure field along the upstream region, i.e.the entrance region. It is in this spirit of the Graëtz process improvements that we expanded this study. It is a question of utilizing the advantages of the Graëtz heat exchanger with converging plates by a deep characterization of its thermal properties.

In the future work, it is important to consider both the inclination angle α and the aspect ratio Γ to improve the modified Graëtz heat exchange system. In addition, we plan to perform a three-dimensional study of the Graëtz process, for which we expect to have more information on the classical and modified Graëtz heat exchangers.

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