

INHOMOGENEOUS AND ANISOTROPIC FIELD THEORY OF GRAVITATION AND ELECTROMAGNETISM: FUNDAMENTALS OF RELATIVISTIC UNIFICATION

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A relativistic field theory of inhomogeneous and anisotropic matter and radiation with a mathematical manifold structure should account for an impersonal picture of the world and contact with physical observables. To incorporate in the theory the interacting nature of microscopic and macroscopic constituents presents philosophical, physical, and mathematical issues. Such a field theory of gravitation and electromagnetism can be motivated by averaging the constituents of fine-grained space-time regions, but a gauge-independent theory of the averaged intrinsic tensor fields defined on a coarse-grained mathematical manifold with structure can only be deduced and closure of the fields established by a relativistic theory of constitutive equations.

Introduction.

The Universe is neither homogeneous nor isotropic on scales less than about 200 Mpc¹ (Fig. 1), as attested by numerous observations of spectra of different cosmic objects [1]. These objects are gaseous and dust-laden nebulae where the stars are forming and producing galaxies and galaxy clusters, cosmic webs connecting the walls of clusters, voids in the webs where the “dark matter” and “dark energy” may reside and cause the expansion of the universe, stars collapsing into black holes and neutron stars, exploding stars producing heavy elements, etc. (Fig. 2).

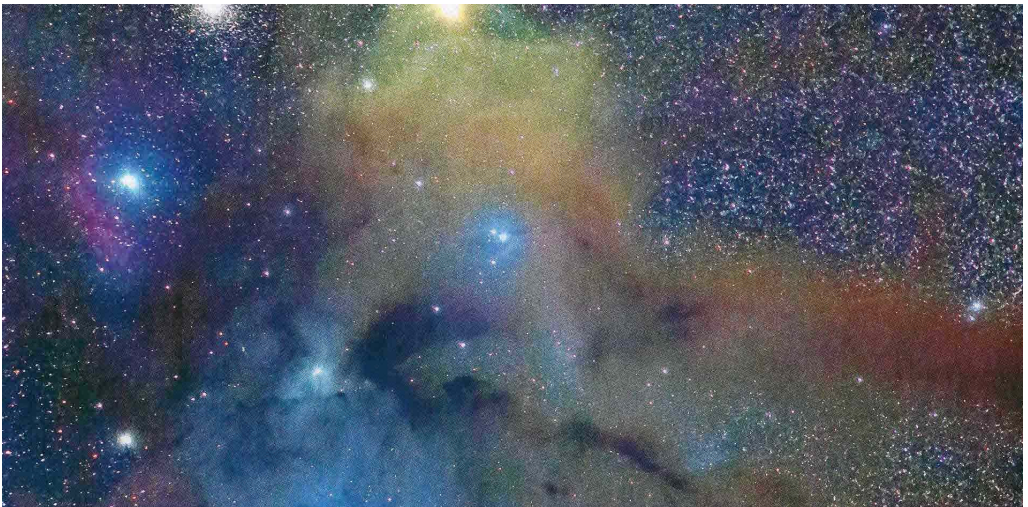


Fig. 1. The Rho Ophiuchi Region of the universe is about 500 light-years away from the Earth. This inhomogeneous and anisotropic region of the cosmos is complex and rich with stars, nebulae and global clusters of galaxies, and can be viewed with large binoculars under dark skies.

¹1pc \sim 3.26 light-years.

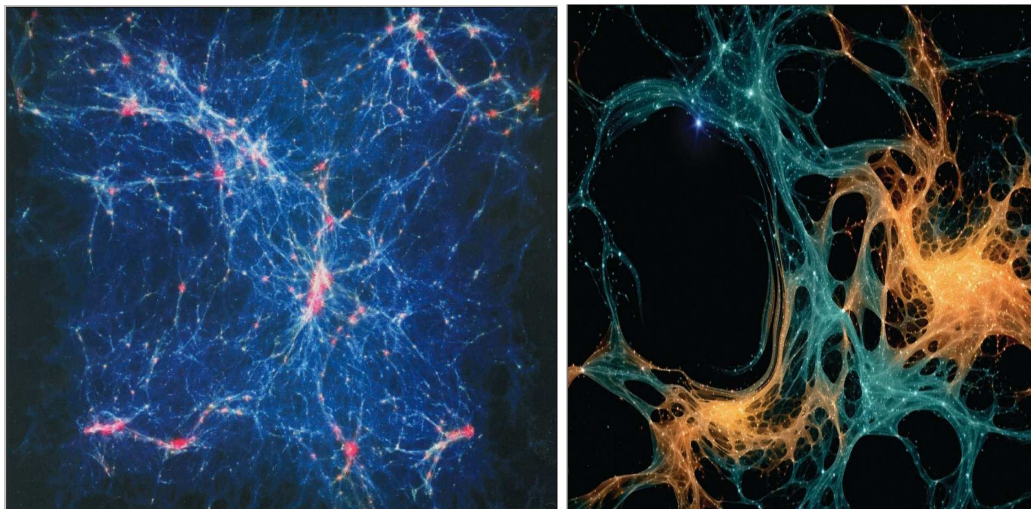


Fig. 2. The Swiss cheese cosmos. (left) Distribution of galaxies embedded in the Lyman- α cosmic web structure. The web consists of tenuous threads of gaseous and dusty matter extending between the clusters of galaxies. (right) The cosmic voids in the web may contain the dark matter and dark energy causing the expansion of the Universe.

The Milky Way contains our solar system and is a spiral galaxy about 1000 light-years thick, 90 000 light-years in diameter, and contains more than 100 000 billion stars. Some of these stars are old as the Universe whose age is estimated to be 14 billion years. This galaxy has several smaller satellite galaxies which are gravitationally bound, and these galaxies are embedded in large-scale structures of cosmic webs. The matter in the Universe was formed from the clouds produced from the Big Bang explosion, where the molecular hydrogen, helium, and heavier elements have been gravitationally binding to form gaseous, liquid and solid bodies, such as planets, stars, black holes, and neutron stars of various sizes. Stars evolve by changes of compositions through various types of fusion reactions [2]; first by burning hydrogen, then helium, and progressively heavier elements lithium, boron and carbon in low-mass stars and heavy elements in large-mass stars. Through the gravitational collapse the low-mass stars eventually evolve into dense bodies, such as white dwarfs, neutron stars, and black holes. From a black hole the matter can escape only by the quantum mechanical tunneling and by the aid of very strong magnetic fields produced from the swirling motions of matter and radiation surrounding and feeding the black hole (Fig. 3). Large-mass stars produce heavy elements through the catastrophic events of supernova nucleosynthesis.

The *strong cosmological principle* assumes that at all scales the Universe is homogeneous (at any time t the Universe looks the same at any place) and isotropic (at any time t the Universe looks similar in all directions), and that it has no center or edge. But the observations with powerful earth and space-based telescopes suggest that only the *weak cosmological principle* is valid, where the homogeneity and isotropicity² is associated at scales greater than about 200 Mpc [1, 3]. The Hubble Space Telescope, the James Webb Space Telescope, and many smaller special purpose instruments have been

²*Homogeneity* is interpreted such that the space-time manifold can be sliced into a one-parameter family of space-like hypersurfaces Σ_t which are homogeneous, whereas the *isotropicity* that an observer sees the Universe locally isotropic at any time t .



Fig. 3. The spiral galaxy NGC 1097 is about 45 million light-years away from the Earth and is severely interacting with two companion galaxies. At its center resides a supermassive black hole with its ring populated with star-forming regions. The spirals of the galaxy extend tens of thousands of light-years beyond the ring of about 5000 light-years in diameter.

providing outstanding high-resolution data of the cosmos from the infrared to the X-ray spectrum. The cosmic microwave background measurements suggest that the Universe is today almost flat or that its four-dimensional *space-time* curvature is nearly equal to zero, and that it has been expanding for the last 14 billion years. These data are necessary to validate the models of the Universe, but they are not sufficient for establishing physical laws of cosmic structures, because such laws must be valid not only for the observers on the Earth but also for any observer in the Universe.

The *standard model of cosmology* is the perturbed Friedmann–Lemaître–Robertson–Walker (FLRW) model (see below) which is based on the cosmological principle and on the Einstein’s Theory of General Relativity [4], and to make this model consistent with observations requires *dark matter* and *dark energy* [1]. The dark matter accounts for the deficiency of observed matter in the Universe and the dark energy for counteracting the gravitational attraction of matter and thus contributes to the accelerating expansion of the Universe. The existence of dark matter and dark energy has not been directly verified, and the inhomogeneous and anisotropic physical models should provide better explanations for the distributions of matter and energy in the Universe.

The Einstein’s deductive fine-grained macroscopic field theory of gravitation and electromagnetism forms the basis of the standard model of the Universe, as well as the models that incorporate the inhomogeneous nature of the Universe obtained by averaging the Einstein’s field equations. All field theories are based on some type of averaging that destroys information somehow accounted through the appropriate macroscopic fields. The current cosmological modeling approach is briefly considered in Section 1, whereas its extension to modeling mixtures of microscopic and macroscopic constituents is described in Section 2.

1. Cosmological modeling based on the theory of general relativity.

1.1. *Einstein's theory of general relativity.* The Einstein's Theory of General Relativity [4] is a homogeneous and isotropic deductive macroscopic field theory of gravitation with the geometry of pseudo-Riemannian 4-dimensional space-time, where the primitive concepts of rods and clocks provide the physical basis of space-time *interval* ds

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (1)$$

where $g_{\mu\nu}$ is the rank=2 *metric tensor* of the space-time geometry and dx^μ is the infinitesimal increment of the arbitrary coordinate $x^\mu = (x^0, x^1, x^2, x^3)$ designating time and three space components³. The metric tensor provides the physical interpretation of the theory, and the interval is posited to be invariant between any two space-time events when measured by no one in particular. The stress and energy of matter fields are expressed by the rank=2 *stress-energy tensor* $T^{\mu\nu}$ which is assumed to be symmetric and covariant divergence-free, i.e.

$$T^{\mu\nu} = T^{\nu\mu}, \quad T^{\mu\nu}_{;\nu} = 0, \quad (2)$$

where the semicolon denotes the covariant derivative. The inertia and gravitational fields belong to the geometric field of space-time, where the stress-energy of matter may or may not exist. This deductive reasoning led Einstein to the next step of coupling the geometry of space-time with its matter and radiation content.

Einstein reasoned that the pseudo-Riemannian space-time should be consistent with the space-time of his Theory of Special Relativity, where the space-time is flat or curvature-free. The curvature of 4-dimensional Riemannian space $R^{\mu\nu\rho\sigma}$ is a rank=4 tensor which does not provide the characteristics required by the stress-energy tensor (Eq. 2), but the contraction of $R^{\mu\nu\rho\sigma}$ with the symmetric tensor $g_{\rho\sigma}$, $R^{\mu\nu\rho\sigma} g_{\rho\sigma} = R^{\mu\nu}$, provides the same rank tensor except that its covariant divergence does not vanish as required by Eq. (2). The tensor

$$G^{\mu\nu} = G^{\nu\mu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}, \quad G^{\mu\nu}_{;\nu} = 0 \quad (3)$$

is, however, consistent with Eq. (2) and $G^{\mu\nu}$ is called the *Einstein tensor*. $R^{\mu\nu}$ is the *Ricci tensor* and $R = R^{\mu\nu} g_{\mu\nu}$ is the *curvature scalar* of the space-time geometry. Einstein then deduced that the space-time geometric field $G^{\mu\nu}$ and the stress-energy field of matter $T^{\mu\nu}$ are related as

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = T^{\mu\nu}, \quad (4)$$

where the geometric field is interpreted to be the carrier (or the consequence) of the inertia and gravitational fields of the matter. The matter field $T^{\mu\nu}$ incorporates the coupling constant $k = 8\pi\Omega$ which is determined from the consistency between the Newtonian and Einsteinian theories of gravitation in the absence of relativistic effects. Ω is the universal gravitational constant.

The Einstein's Eq. (4) form the bridge linking geometrical objects and empirically confirmed physical laws. Because of the required symmetry of the metric tensor⁴, the rank=4 curvature tensor $R^{\mu\nu\rho\sigma}$ of the Riemannian space provides many symmetries and thus the Einstein's field equations provide only 10 partial differential equations which

³This paper employs the metric signature $(-+++)$ and Einstein's summation convention.

⁴ $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{\nu\mu} dx^\nu dx^\mu = g_{\nu\mu} dx^\mu dx^\nu, g_{\mu\nu} = g_{\nu\mu}.$

have 4 constraints provided by the vanishing divergences in Eqs. (2) and (3). The Einstein's equations thus provide six independent equations with 6 unknowns for the metric tensor, which with suitable boundary conditions can be solved for the components of the metric tensor as functions of four independent coordinates of space-time.

In the space-time where the matter is absent, the Einstein tensor

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} \quad (5)$$

does not necessarily vanish. This occurs between the matter objects, where the electromagnetic radiation experiences gravitational effects from the space-time curvature (bending of light around stars). The motion of matter requires curved space-time, whereas the propagation of radiation can occur in both flat and curved space-time. The mass of matter in relativity is not conserved and appears in the stress-energy tensor in the form of mass-energy density.

The symmetric and divergence-free field $G^{\mu\nu}$ requires that the stress-energy field $T^{\mu\nu}$ possesses the same tensor characteristics and that, therefore, the combined *electromagnetic energy tensor* $E^{\mu\nu}$ and *matter stress-energy tensor* $M^{\mu\nu}$ possess the same characteristics,

$$T^{\mu\nu} = E^{\mu\nu} + M^{\mu\nu}, \quad T^{\mu\nu} = T^{\nu\mu}, \quad T_{;\nu}^{\mu\nu} = 0. \quad (6)$$

The matter exhibits electromagnetism (both in its free and bound forms) and its motion affects its structure and space-time field geometry (or the space-time field affects the radiation and structure of matter), and led Einstein, Weyl, and Eddington [5] (and subsequently other individuals) to propose different theories of relativistic unification of gravitation and electromagnetism. The ideal and dissipative homogeneous and isotropic hydrodynamic models of the Universe were incorporated into the theory through the stress-energy tensor $M^{\mu\nu}$ of matter, and the Maxwell's electromagnetic energy-momentum tensor $E^{\mu\nu}$ through the *electromagnetic tensor potential* $F^{\mu\nu}$, i.e.

$$E^{\mu\rho}g_{\rho\nu} = E_{\nu}^{\mu} = F^{\mu\rho}F_{\nu\rho} - \frac{1}{4}g_{\nu}^{\mu}F^{\rho\sigma}F_{\rho\sigma}. \quad (7)$$

The Einstein's Eq. (4) with conditions (6) are consistent with the *cosmological principle* by which the Universe is homogeneous and isotropic at suitably large scales, but this is not true below the scales of less than about 200 Mpc. Another issue of this theory is that it does not account for the apparent existence of additional matter and energy in the Universe.

1.2. Standard model of cosmology. The Standard Model of Cosmology (SMC) is the Friedmann–Lemaître–Robertson–Walker (FLRW) model based on the cosmological principle with the line element of a free-falling observer

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}(|\mathbf{x}|)dx^i dx^j, \quad (i, j) = (1, 2, 3), \quad (8)$$

where γ_{ij} is the space metric, t is the cosmic time, and \mathbf{x} denotes spherical coordinates of the space. The stress-energy tensor $T^{\mu\nu}$ of SMC is that of the perfect fluid with energy density ρ and pressure p ,

$$T^{\mu\nu} = 8\pi\Omega [(\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}], \quad (9)$$

where u^{μ} is the fluid velocity in the coordinate frame x^{ν} .

The SMC cosmological model is, however, deficient as it cannot adequately account for the matter and energy contents of the Universe, and thus for the Universe's observed expansion. For the static Universe, Einstein proposed the addition of a *cosmological constant* Λ to Eq. (3), but then he retracted this idea when Hubble discovered that the Universe is expanding [6]. The discovery of the acceleration of the expansion of the Universe in 1998 [7] resurrected the cosmological constant which now appears in the so-called *concordance cosmological model* (Λ CDM), i.e.

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} + \Lambda g^{\mu\nu} = T^{\mu\nu}. \quad (10)$$

The Λ CDM model accounts for the Cosmic Microwave Background (CMB)⁵, large-scale structure of the Universe, ordinary baryonic matter and energy, , insufficiently, for the missing matter and energy called the dark matter and dark energy, respectively. The key issue associated with modeling the Universe is: What is the appropriate theory of relativity that can explain the current observations and guide the astrophysicists toward the discovery of new astrophysical phenomena?

1.3. The averaging paradigm. A possible explanation of dark matter and dark energy may come from the inhomogeneous nature of the Universe obtained from averaging the fine-grained (or natural) macroscopic tensor field equations of the Einstein's Theory of General Relativity. There is an extensive literature on this subject and has produced the *averaging problem*, where only the covariant averaging is acceptable. Such an averaging transforms the fine-grained or *background* cosmological fields to the coarse-grained fields which are associated with physical observables, and to the fields associated with non-commutativity of the averaging operators [8, 9]. The tensor fields of the Einstein's equations reside within the Einsteinian fine-grained manifold structure, whereas the coarse-grained fields belong to a mapped manifold with a different structure. The relation between these two manifolds depends on the nature of the averaging space-time region (volume or hypersurface) and on its constituents (matter and radiation) and their interactions.

The averaging process produces a loss of information and additional macroscopic fields must be employed to deal (directly or indirectly) with this loss. This loss causes the entropy increase which is reflected in the balance of energy and momentum and thus may be identified as a new form of energy and matter. In the physics literature, this loss is interpreted as "dark energy" and "dark matter" [10], but it could be interpreted as the macroscopic energy and matter arising from the generalization of quasi-reversible Einsteinian (ideal macroscopic) description of nature to irreversible entropy producing description, where the structured cosmic objects are parts of this description.

In the theory of relativity, the transfer of information is limited by the local speed of light in the intervening medium between the events, and an improper averaging can violate the *principle of causality* which is fundamental in physical science. The observations of cosmic phenomena are usually conducted over certain periods of time and regions of space and it appears that it is advantageous to average over the space-time, but this may seriously violate the principle of causality and impede reduction of the theory to its Galilean space-time form. The non-relativistic Theory of Structured Multiphase Mixtures [11, 12] is based on the space-averaging, and this theory should be consistent with a more general relativistic form when the information is assumed to propagate at infinite speed.

⁵Radiation from the creation of the Universe about 13.7 Gyrs ago.

An effective inhomogeneous and anisotropic cosmological theory should account for mixtures of photons and dissolved and exsolved chemical species in macroscopic bodies, such as dust-laden nebulae, galaxies and galaxy clusters, cosmic webs, planets, stars, etc., where mutual interactions are essential for the formations of structures. To distinguish the microscopic from the macroscopic bodies it is convenient to designate the dissolved and sufficiently small (such as dust) chemical bodies as *components* and larger macroscopic bodies as *phases*, where each specie can possess its own kinematic, dynamic, and thermodynamic properties. The current cosmological modeling approach is based on the single-phase multicomponent modeling paradigm which by definition is homogeneous and isotropic and cannot be effectively implemented for studying complex cosmic structures which possess different velocities and accelerations.

The theory proposed in the following section is built from the notion of infinitesimal parallel displacement which is postulated to be significant to the relation structure of the cosmic world. This *fundamental axiom* is essential for translating the relation-structure into mathematical expression from which its consequences can be established. In particular, it is essential that a generalized theory of gravitation and electromagnetism is independent of any particular gauge-system and that the key tensors of the theory must not only have their usual properties of transformations of coordinates but also be unaltered by any transformation of the gauge-system. We will call such tensors the *intrinsic tensors* and designate them with bars over the letters.

2. Inhomogeneous and anisotropic field theory of gravitation and electromagnetism.

2.1. Coarse-grained interval. The cosmological modeling approach considered in this section is different from the existing approaches in that it involves the interactions of fields with *microscopic and macroscopic* qualities at each point \mathbf{x}^μ of a coarse-grained mathematical space-time manifold. Although there are no rules for representing physical quantities, it is necessary that these quantities have (direct or indirect) observational qualities which may and may not be associated with every observer of cosmic objects. The tensor fields of all components and phases of this manifold are, therefore, differentiable and endowed with phasic metrics $g_{\mu\nu}^{(\alpha)}$ and infinitesimal phasic intervals $ds^{(\alpha)}$ such that the observational coarse-grained interval ds is defined as

$$ds^2 = \sum_{\alpha} ds^{(\alpha)2} = \sum_{\alpha} g(x)_{\mu\nu}^{(\alpha)} dx^\mu dx^\nu, \quad (11)$$

where the superscript (α) is not a tensor index but denotes that the quantity pertains to the phase α . Here it is assumed that each phase of the mixture (plasma, gas, liquid, solid) is chemically uniform or consists of different dissolved chemical components, and that a particular phase can be further split into macroscopic objects of different sizes which increase the number of phases in a region of space-time under observation⁶. The symbol \sum_{α} represents the sum of all phases. The metrics $g_{\mu\nu}^{(\alpha)}$ must be tensors and chosen in such a manner that the interval ds corresponds to the *physical* interval and therefore to unique proper time for all constituents of the mixture..

The representation of the coarse-grained interval in Eq. (11) requires a more complex

⁶The assumption that all dissolved chemical components in a continuum particle of the mixture have the same particle velocity is common in the works of Newtonian continuum mechanics, but this assumption can be relaxed at the expense of complicating the theory.

geometry which accounts for stretching and twisting of space-time. For example,

$$ds^2 = \sum_{\alpha} ds^{(\alpha)2} = \sum_{\alpha} g(x)_{\mu\nu}^{(\alpha)}(x) dx^{\mu} dx^{\nu} + f(x) \sum_{\alpha} g(y)_{\mu\nu}^{(\alpha)} dy^{\mu} dy^{\nu} \quad (12)$$

represents the product of geometry x and warped geometry y , and reduces to the Friedmann–Lemaitre–Robertson–Walker interval of Eq. (8). This interval pertains to the large-scale model of the Universe which is homogeneous and isotropic⁷ and expands with the cosmic time function $f(t) = a(t)^2$.

The tensor fields of the (coarse-grained) theory can be motivated through their associations with the fields of the fine-grained Einsteinian manifold, but in our deductive theory we assume that all the tensor fields are (superimposed) space-time averages defined on a mathematical manifold. The averaging is an inductive process which can only *motivate* a deductive theory.

2.2. Generalized theory of gravitation and electromagnetism. The relativistic particles of the continuum form a congruence of worldlines in the background space-time. At each time we define a fixed (from the observer’s point of view) hypersurface which intersects a subset of these worldlines. The averaged fields of this hypersurface produce a point in a new space-time and this process is continued until a coarse-grained space-time manifold is built with phasic metrics $g_{\mu\nu}^{(\alpha)}$ and intervals $ds^{(\alpha)}$ for use in the observational coarse-grained interval ds . This is the motivating side of the theory.

The generalized Einstein field equations for each phase α of the continuum are then constructed from the rank=4 intrinsic curvature tensor $\bar{R}_{\mu\rho\nu}^{(\alpha)\epsilon}$ and intrinsic stress-energy (matter and electromagnetic) tensor $\bar{T}_{\mu\rho\nu}^{(\alpha)\epsilon}$, or from their contracted forms $\bar{R}_{\mu\nu}^{(\alpha)}$ and $\bar{T}_{\mu\nu}^{(\alpha)}$

$$\bar{R}_{\mu\rho\nu}^{(\alpha)\epsilon} = \bar{T}_{\mu\rho\nu}^{(\alpha)\epsilon}, \quad \bar{R}_{\mu\nu}^{(\alpha)} = \bar{T}_{\mu\nu}^{(\alpha)}. \quad (13)$$

The former form implies that all of the components of $R_{\epsilon\mu\nu\rho}^{(\alpha)}$ will be used to determine the metric tensor $g_{\epsilon\mu\nu\rho}^{(\alpha)}$ and that none will be available to set the boundary conditions to solve the partial differential equations of the theory. Thus, only the second form in Eq. (13) allows us to investigate the physical phenomena, but the first form is necessary to express some absolute properties of the structure of space-time manifold.

The phasic tensors $\bar{R}_{\mu\nu}^{(\alpha)}$ and $\bar{T}_{\mu\nu}^{(\alpha)}$ are, in general, *non-symmetric tensors* of rank=2 and the metric tensor $g_{\mu\nu}^{(\alpha)}$ is arbitrary symmetric tensor satisfying the invariance of the interval $ds^{(\alpha)2} = g_{\mu\nu}^{(\alpha)} dx^{\mu} dx^{\nu} = g_{\nu\mu}^{(\alpha)} dx^{\nu} dx^{\mu}$ appearing in Eq. (11)⁸. The intrinsic curvature tensor $\bar{R}_{\mu\rho\nu}^{(\alpha)\epsilon}$ and its contracted intrinsic form $\bar{R}_{\mu\nu}^{(\alpha)}$ are obtained from the non-symmetric *phasic connection* $\Gamma_{\mu\nu}^{(\alpha)\epsilon} \neq \Gamma_{\nu\mu}^{(\alpha)\epsilon}$,

$$\bar{R}_{\mu\nu\sigma}^{(\alpha)\epsilon} = \frac{\partial}{\partial x^{\sigma}} \Gamma_{\nu\mu}^{(\alpha)\epsilon} - \frac{\partial}{\partial x^{\nu}} \Gamma_{\sigma\mu}^{(\alpha)\epsilon} - \Gamma_{\sigma\mu}^{(\alpha)\rho} \Gamma_{\nu\rho}^{(\alpha)\epsilon} + \Gamma_{\nu\mu}^{(\alpha)\rho} \Gamma_{\sigma\rho}^{(\alpha)\epsilon}, \quad (14)$$

$$\bar{R}_{\mu\nu}^{(\alpha)} = \frac{\partial}{\partial x^{\rho}} \Gamma_{\nu\mu}^{(\alpha)\rho} - \frac{\partial}{\partial x^{\nu}} \Gamma_{\rho\mu}^{(\alpha)\rho} - \Gamma_{\delta\mu}^{(\alpha)\rho} \Gamma_{\nu\rho}^{(\alpha)\delta} + \Gamma_{\nu\mu}^{(\alpha)\rho} \Gamma_{\delta\rho}^{(\alpha)\delta}. \quad (15)$$

This connection is not assumed to be a tensor and allows for an infinitesimal displacement of the vector $V^{(\alpha)\rho}$ at the space-time point P (with the coordinates x^{μ}) to be

⁷In homogeneous mixtures all components and phases constituting the particles of mixtures propagate at the same mixture velocity.

⁸The proper times of the constituents of the mixture may not be accurately measurable and therefore the metrics of constituents should not be a priori assumed to be symmetric.

carried by parallel displacement to a point P' (with the coordinates $x^\mu + dx^\mu$) infinitely near to P . The general change of $V^{(\alpha)\rho}$ is

$$dV^{(\alpha)\mu} = \Gamma_{\nu\rho}^{(\alpha)\mu} V^{(\alpha)\rho} dx^\nu. \quad (16)$$

Both V^ρ and dx^ν are infinitesimals and $\Gamma_{\nu\rho}^{(\alpha)\mu}$ represents 64 coefficients of curved 4-dimensional space-time. It is the *fundamental axiom* that out of the whole group of displacements from P' we can select one $V^\rho + dV^\rho$ which is *equivalent* to the displacement V^ρ at P , and Eq. (16) thus accounts for the relation-structure which is supposed to be comparable with all observers [5]. The geometrical representation of relations leads to space-time locations, and the parallel displacement axiom to comparability of proximate relations. The space and time are approximate conceptions for ordering the events.

The affine geometry condition assuming that the connection is symmetric ($\Gamma_{\nu\mu}^{(\alpha)\rho} = \Gamma_{\mu\nu}^{(\alpha)\rho}$) provides that the four points (ABCD) of the parallelogram are structurally related through the equivalence of AB to CD and AC to BD. This condition may result from the statistically-averaged background space-time as in the Einstein's theory, and if it is not fulfilled as in generalized Einstein's theory it will produce new phenomena. In the former situation, the connection $\Gamma_{\nu\mu}^{(\alpha)\rho}$ is obtained from 6 symmetric metric tensor field components $g_{\mu\nu}^{(\alpha)}$ and 10 antisymmetric length displacement field components $K_{\mu\nu\rho}^{(\alpha)}$ of each phase. $K_{\mu\nu\rho}^{(\alpha)}$ appears from the displacement of the phasic length $\ell^{(\alpha)2} = g_{\mu\nu}^{(\alpha)} D^{(\alpha)\mu} D^{(\alpha)\nu}$ of the parallelly displaced vector field $D^{(\alpha)\mu}$ and it vanishes when the space is free from fields. Thus,

$$\begin{aligned} d(\ell^{(\alpha)2}) &= \left(\frac{\partial g_{\mu\nu}^{(\alpha)}}{\partial x^\rho} - g_{\epsilon\nu}^{(\alpha)} \Gamma_{\sigma\mu}^{(\alpha)\epsilon} - g_{\mu\epsilon}^{(\alpha)} \Gamma_{\sigma\nu}^{(\alpha)\epsilon} \right) D^{(\alpha)\mu} D^{(\alpha)\nu} dx^\rho \\ &= K_{\mu\nu\rho}^{(\alpha)} D^{(\alpha)\mu} D^{(\alpha)\nu} dx^\rho. \end{aligned} \quad (17)$$

The geometry pertaining to the phasic field Eqs. (13) is associated with both the gravitational and the electromagnetic fields and does not include the cosmological constant Λ as Eq. (10), because the inhomogeneity and anisotropy of coarse-grained space-time are built into the intrinsic curvature and intrinsic tensors. These tensor fields should account for the apparent absence of matter and energy in the Universe and can be closed by employing a relativistic theory constitutive equations, sixteen variables associated with the gravitational and length displacement metrics, fields associated with the structure of the constituents of the mixture, and the invariants of curvature and stress-energy tensors.

3. Conclusion.

The Einstein's Theory of General Relativity requires extensions for applications to inhomogeneous and anisotropic gravitational and electromagnetic phenomena. Such extensions can be motivated by suitable averaging of fine-grained structure of the Einstein's equations, but the averaging cannot be used to establish the foundations of a generalized inhomogeneous and anisotropic deductive field theory which employs the tensor analysis on manifolds. The generalized theory of gravitation and electromagnetism described in the paper involves mixtures of microscopic and macroscopic components, which define the tangent space of the coarse-grained manifold. The space-time field of the theory is represented by the intrinsic curvature tensor and the matter and radiation fields by the intrinsic stress-energy tensor for each phase of the mixture. A relativistic theory of constitutive equations provides closure of the fields, and is presented in another contribution.

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